ICMIEE18-256

Numerical Simulation and Analysis of Supersonic Flow over a Circular Cylinder

Shawon Kumar Saha
Department of Mechanical Engineering, Bangladesh University of Engineering & Technology, Dhaka-1000, BANGLADESH

ABSTRACT
The purpose of this study is to investigate a two dimensional supersonic flow over a circular cylinder numerically. The calculations are performed on a boundary fitted co-ordinate system. Time dependentNavier-Stokes equations is used to evolve the correct steady-state solution. The analysis is conducted by assuming a rigid circular cylinder with a wide range of Mach number (2,4,6,7) and two different temperatures (300K and 373K) by Ansys Fluent We consider air as calorically perfect gas, with constant Prandtlnumber and Sutherland's law for the viscosity. The two dimensional Navier-Stokes equations for a unsteady flow, with no body forces, no mass diffusion are solved. Flow fields are obtained. The pressure along the entire surface is computed over a wide range of Mach number and two different temperatures. The temperature variation due to dissipation of kinetic energy has been analyzed numerically.

Keywords: Computational fluid dynamics, Supersonic Flow, Numerical Investigation, Sutherland's Law, Circular Cylinder

1. INTRODUCTION

The flow field of a velocity greater than that of a sound, In which a circular cylinder is mounted, is very complicated, because a detached shock wave is originated ahead of the cylinder and behind it a rotational flow filed of mixed type, that is, containing super and sub-sonic regions appear.It is quite important to analyze the Supersonic flow over various objects as it has a great importance in aerodynamics and theoretical aspects in fluid mechanics. In order to evaluate such phenomena, the 2-D viscous flow over a calorically perfect gas (we consider air with constant Prandtl number and Sutherland's law for viscosity) a wide range of Machnumber over a cylinder was simulated. Flow motion is represented by the compressible Navier-Stokes equations, and they are solved with Finite Volume techniques. Among the different approaches existing in the literature for the numerical fluxes discretization, a hybrid initialization is preferred.This type of numerical schemes preserve kinetic energy of turbulent scales. Hybrid initialization is a collection of recipes and boundary interpolation methods. The pressure along the entire surface has been computed over a wide range of Mach numbers and two different temperatures. The temperature variation due to dissipation of kinetic energy has been analyzed numerically. The two dimensional Navier-Stokes equation for non-steady flow, with no-body forces, no volumetric heating and no mass diffusion are solved.

Deng-Pan *et al.*[1] investigated the flow structure of a supersonic flow over a cylinder by method of flow visualization and with nanoparticle-based planar laser scattering in a supersonic quiet wind tunnel at Mach number of 2.68. Based on the time correlation of NPLS Images the time space evolutionary characteristics of the structure in the supersonic flow over a finite cylinder has
Been studied and the characteristics of the structure in the Flow direction are obtained.

Poplavskaya*et al* [2] studied the supersonic flow around a stream wise aligned cylinder with a frontal gas permeable insert made of a high porosity cellular material are presented. The computed results were compared with the data of wind tunnel experiments performed in T-327B supersonic continuous flow wind tunnel at the flow Mach number 4.85.Experimental and computed data for the normalized drag co-efficient of the model versus the normalized length of the porous insert and versus the Reynolds number are analyzed.

Bashkin*et al*[3] investigated the supersonic flow of perfect gas past a circular cylinder with an isothermal surface at the Mach number 5 and Reynolds Number ranging from (30 to 500000). He showed that two branches of numerical solution of the problem can exist. Pressure coefficient distribution on the rear of an isothermal surface at Mach number 5 was analyzed. Velocity profile in the plane of symmetry downstream of an isothermal cylinder at Mach number 5 was also analyzed.

Rajani*et al* [4] focused on the two and three dimensional flow past a circular cylinder in different flow regimes. He used an implicit pressure based finite volume method for time accurate computation of incompressible flow. Both experimental measurements and numerical computations were done. Temporal variation of lift coefficient on time and frequency domain at different Reynolds number were analyzed also.

In the present study a circular cylinder is studied at two different temperature (300K and 373K) at a wide range of Mach No. (2, 4, 6, 7) and pressure and temperature distribution are computed. This study also includes the comparison of static temperature and pressure distribution for two different temperatures at a fixed Mach number. This is done to analyze the effect of stagnation temperature.

*Shawon Kumar Saha. Tel.: +88-01823348447
E-mail addresses: shawon14106@gmail.com

2. COMPUTATIONAL METHOD

2.1 Mathematical Model

This problem is considered with interesting fluid phenomena. The most complete model available for the flow of air is the Navier-Stokes equations. The advantage of using time-dependent Navier-Stokes approach is its inherent ability to evolve to the correct steady-state solution. However, they represent a model not a physical truth. They represent the three conservation laws. Neglecting body Forces and volumetric heating and mass diffusion the two dimensional forms of the Navier-Stokes equations are

Continuity Equation:

$$\frac{\partial \rho}{\partial t} + \frac{1}{r}\frac{\partial}{\partial r}(\rho r u_r) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta) = 0 \quad (1)$$

r- Momentum:

$$\frac{\partial(\rho u_r)}{\partial t} + \frac{\partial}{\partial r}(\rho u_r^2 + P) + \frac{1}{r}\frac{\partial}{\partial r}(r\tau_{rr}) + \frac{1}{r}\frac{\partial}{\partial \theta}(\rho u_\theta u_r + \tau_{\theta\theta}) + \frac{1}{r}(\tau_{\theta\theta} - u_\theta^2) = 0 \quad (2)$$

θ- Momentum:

$$\frac{\partial(\rho u_\theta)}{\partial t} + \frac{1}{r}\frac{\partial}{\partial \theta}(u_\theta^2 + P + \tau_{\theta\theta}) + \frac{\partial}{\partial r}(\rho u_r u_\theta) + \frac{1}{r^2}\frac{\partial}{\partial r^2}(r^2\tau_{r\theta}) = 0 \quad (3)$$

Energy Equation:

$$\frac{\partial(\rho c T)}{\partial t} + u_r\frac{\partial(\rho c T)}{\partial r} + \frac{u_\theta}{r}\frac{\partial(\rho c T)}{\partial \theta} = \varphi_v + k\{\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \frac{\partial^2 T}{\partial \theta^2}\} \quad (4)$$

Where t, ρ, u_r, u_θ, P are the time,density,velocity in r-direction and θ-direction respectively and $\tau_{r\theta}, \tau_{rr}, \tau_{\theta\theta}$ are the stresses.This formsa four basic equations (Ref [5]). For solving these equations four additional equations used are the equation of state for a perfect gas.

These additional four equations are

$$\tau_{rr} = -\mu\{2\frac{\partial u_r}{\partial r} - \frac{2}{3}(\nabla.\vec{u})\} \quad (5)$$

$$\tau_{\theta\theta} = \mu\left\{2\left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right) - \frac{2}{3}(\nabla.\vec{u})\right\} \quad (6)$$

$$\tau_{r\theta} = \tau_{\theta r} = -\mu\left\{r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}\right\} \quad (7)$$

And φ_v is the dissipation rate of energy and it is represented as

$$\varphi_v = \mu\left[2\left\{\left(\frac{\partial u_r}{\partial r}\right)^2 + \left(\frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}\right)^2\right\} + \left\{r\frac{\partial}{\partial r}\left(\frac{u_\theta}{r}\right) + \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right\}^2\right] \quad (8)$$

Where,

$$\nabla.\vec{u} = \frac{1}{r}\frac{\partial}{\partial r}(r u_r) + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta}$$

2.2 Solver Settings

The solver setting used was ANSYS Fluent. The settings are given in Table 1.

Table 1 Solver, Viscous model

Function	Option
Model	k-ω (2 eqn),Energy
K-ω model	SST
Turbulent viscosity	None
Options	Curvature Correction
Solver	Density-based
Space	2D
Velocity Formulation	Absolute
Formulation	Implicit
Flux Type	Roe-FDS
Gradient	Least Square Cell Based
Flow	Second Order Upwind

3. Computational Domain

As shown in Figure 1 the computational domain was discretized with a user defined structured mesh. Maximum face size was taken (5.3852e-003 m) with a growth rate of 1.20 and defeaters size of (2.6926e-005 m) where the minimum face size was considered as (5.382e-005).Element Order was taken as quadratic. Free mesh face type was kept quadratic or triangular. The mesh around the cylinder was made denser to ensure high accuracy.

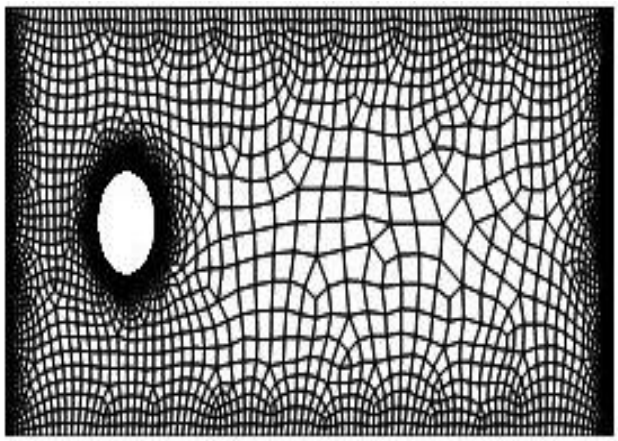

Fig.1 Mesh of the complete domain

4. Boundary Conditions

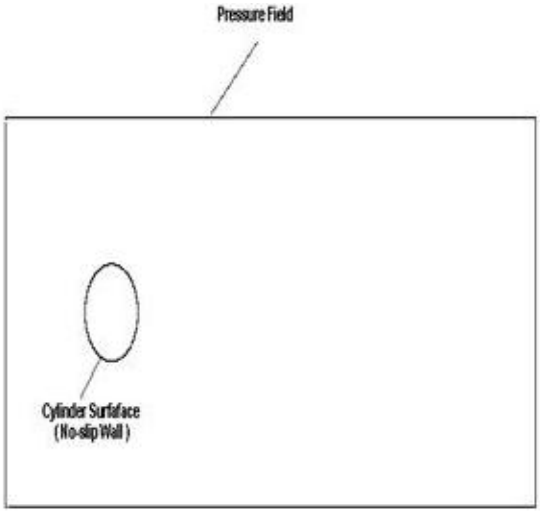

Fig.2 Boundary Conditions

As shown in Figure 2 the boundary of the domain is consisted of a pressure far filed. The cylinder wall was taken to be no slip wall and standard roughness model with a roughness constant of 0.5. The flowing fluid was air with constant density, specific heat thermal conductivity. Sutherland Law is used for the viscosity of flowing fluid. Four different cases was considered by varying the Mach number at two different temperature of 300K and 373K. The Mach number used were 2, 4, 6, 7.

Table 2 Cases of Flow

Cases	Mach Number	Temperature
Case-1	2	
Case-2	4	300K 373K (for all cases)
Case-3	6	
Case-4	7	

5. RESULT AND DISCUSSIONS

The static surface pressure distribution in the entire flow field was plotted in Fig (3 & 4) as a function of distance from the inlet of the flow field. Static Surface pressure was computed for Mach numbers (2, 4,6,7) and two different temperature of 373K and 300K. As shown in (Figure 3) initially oscillators were observed showing higher increase in pressure in the region near the cylinder wall. The result was relatively lower density and hence thick boundary layer therefore it create a strong leading edge shockwave thus increase in the pressure within the shock layer. However, the oscillator disappeared past the cylinder wall and stable result was obtained. With the increase in Mach number there was a consequent increase in the static surface pressure. The pressure in both the cases was gradually decreasing after the cylinder wall along the entire field and result in almost stable value.

It was seen that the pressure was maximum at the stagnation point and decreases as a function of distance away from the stagnation point- a variation that we would expect[6].After reaching the peak value the static pressure decrease and approached at a steady value.

The temperature distribution in the entire flow field was computed.The static temperature over the entire flow field for the inflow velocity for different Mach numbers and two different temperature were plotted in Fig (5 & 6). It was observed that due to the formation of shock layer the temperature increased as shown by the plot. Again in this case like the Pressure distribution profile the temperature was decreased after a certain point, actually after passing the cylinder wall and comes to almost constant temperature.It was seen in (Figure5 and 6) like pressure distribution profile here also with the increased of Mach numbers the temperature was increasing. It can be seen that the static temperature was higher for stagnation temperature of 373Kthan the stagnation temperature of 300K. Like the static pressure, static temperature also decreased and approached at a steady value.

It can be seen that similar curves was found for the static temperature distribution. Here the static temperature was maximum at the stagnation point and lately decreased as a function of distance away from the stagnation point.

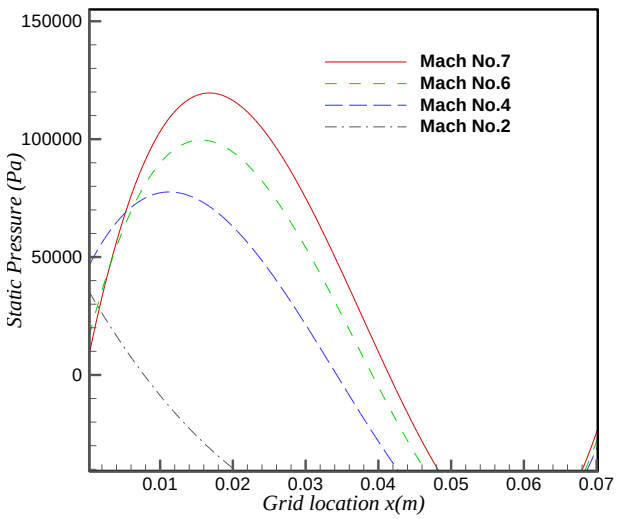

Fig. 3 Variation of Static Surface Pressure at 373K for different Mach numbers.

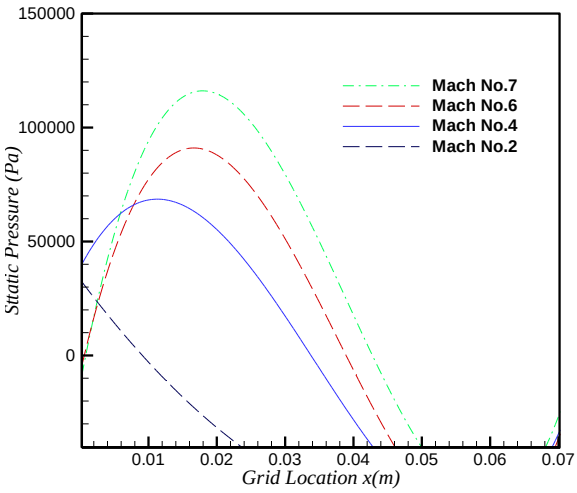

Fig. 4 Variation of Static Surface Pressure at 300K for different Mach numbers.

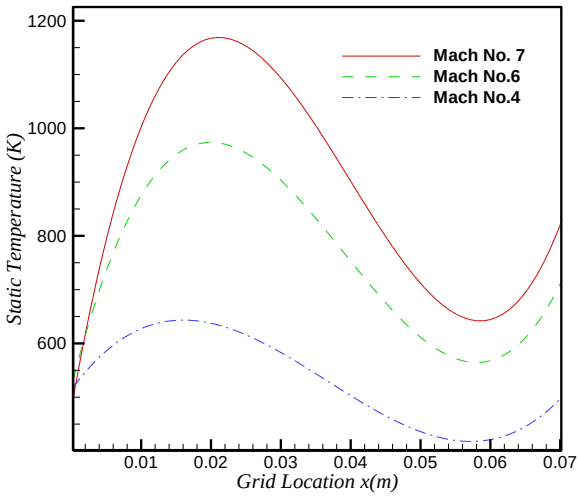

Fig. 5 Variation of Static Temperature at 373K for different Mach numbers.

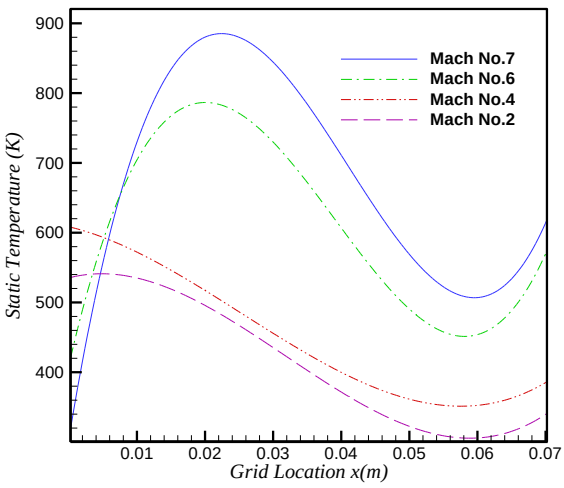

Fig. 6 Variation of Static Temperature at 300K for different Mach numbers.

The comparison of static pressure and static temperature for different temperature at Mach No. 6 were computed. The variation for these two temperatures(373K and 300K) were plotted against the grid location in (Figure 7 and 8). It can be seen that static temperature and static pressure was higher for 373K than 300k. If stagnation temperature increased static pressure and temperature will increase [7]. Similar results were found in this study.

The relation between stagnation temperature and static temperature is

$$\frac{T_0}{T} = 1 + \frac{k-1}{2} M^2 \tag{9}$$

$$Mach\ Number, M = \frac{V}{\alpha} \quad ;$$

Wherethe speed of sound,$\alpha = \sqrt{kRT}$

$$and\ c_p = \frac{kR}{k-1}$$

So, the relation becomes,

$$\frac{T_0}{T} = 1 + \frac{(kR)V^2}{2c_p(kRT)} \tag{10}$$

Andthe relation between static pressure and stagnation pressure is

$$\frac{P_0}{P} = \left\{1 + \frac{(kR)V^2}{2c_p(kRT)}\right\}^{\frac{k}{k-1}} \tag{11}$$

Here P_0, T_0 is the stagnation pressure and temperature where $P\ and\ T$ are the static pressure and temperature and c_p, k are the specific heat at constant pressure and ratio of specific heat. From the above relations it can be observed that if stagnation temperature increased static pressure and temperature will increase.

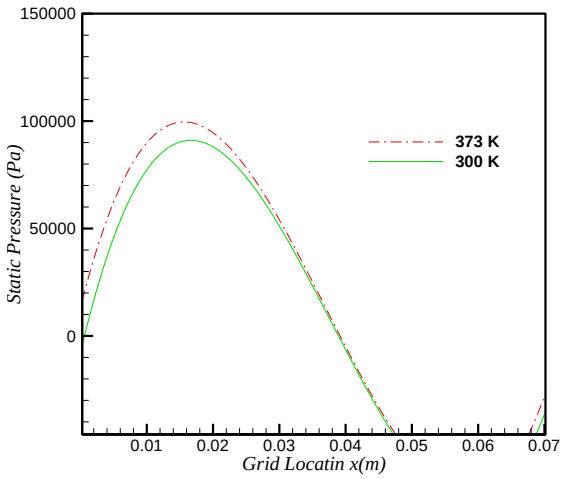

Fig. 7 Variation of Static Pressure for two different stagnation temperature (Mach No. 6).

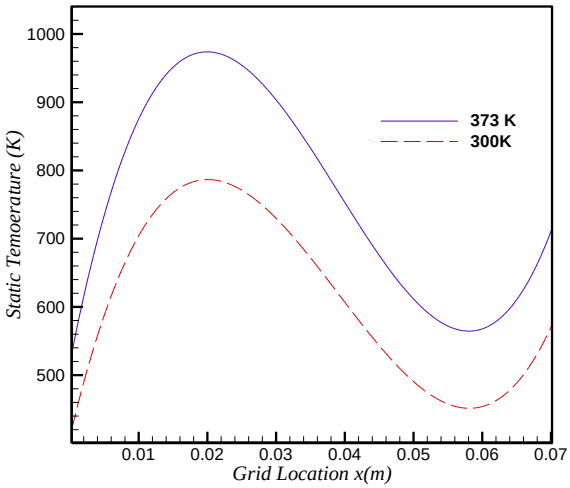

Fig.8 Variation of Static Temperature for two different stagnation temperature (Mach No. 6).

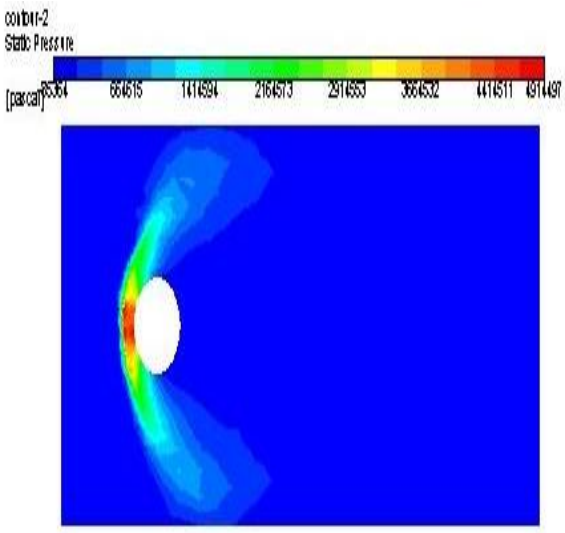

Fig.9 Pressure Contours at Mach No. 6 for flow over a Circular Cylinder (Temperature 373K)

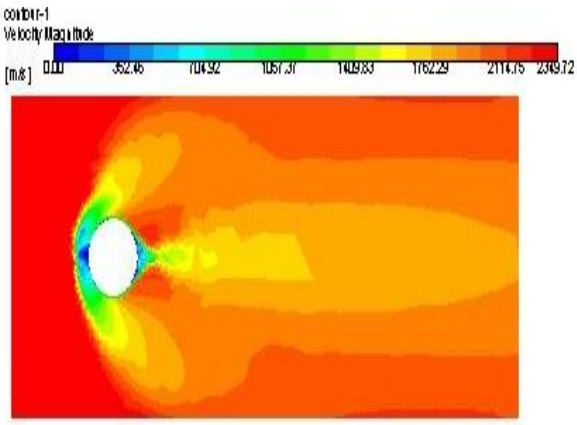

Fig.10 Velocity Contours at Mach No. 6 for flow over a Circular Cylinder (Temperature 373K)

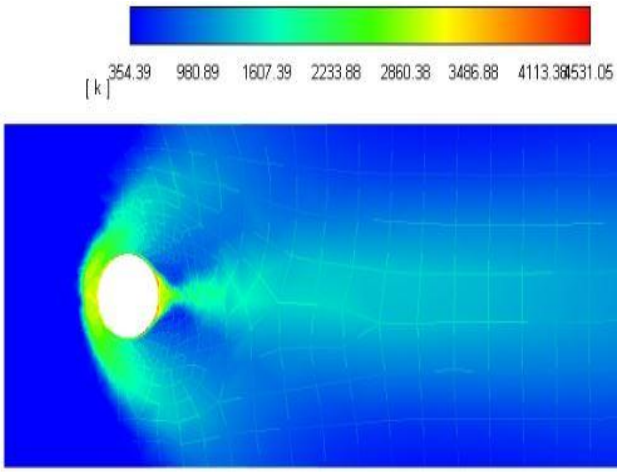

Fig.11 Temperature Contours at Mach No. 6 for flow over a Circular Cylinder (Temperature 373K)

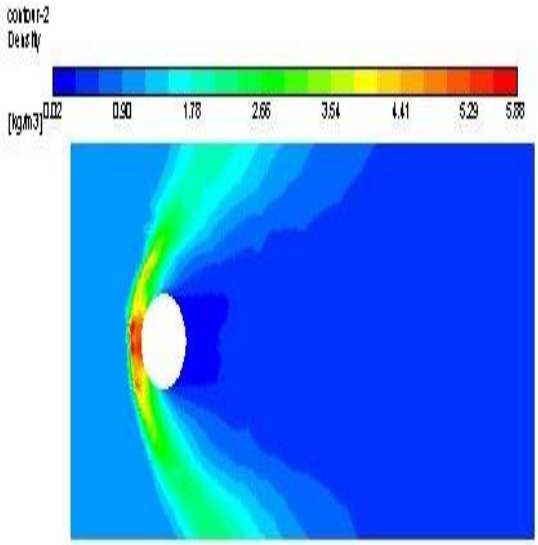

Fig.12 Density Contours at Mach No. 6 for flow over a Circular Cylinder (Temperature 373K)

The velocity, pressure, density and temperature contours were obtained at Mach No. 6 at temperature of 373K. In the pressure contour, it can be seen that, the static pressure upstream the cylinder was higher and gradually decreased downstream the cylinder which we would expect for the pressure distribution of a supersonic flow over any blunt bodies.

In the study of temperature contour, similar results were found which showed the high temperature upstream the cylinder and decreasing temperature downstream the cylinder. Even though similar result were found for density contours it can be observed that density contour had higher gradient than the temperature contour.

Shock waves are formed when a pressure front moves at a supersonic speed and pushes on the surrounding air [8]. At the region where this occurs sound wave travelling against the flow reaches at a point where they could not travel any further upstream and the pressure

progressively builds up in this region. Shock waves are very small region in the gas where the gas properties changes by a large amount. Across the shock wave the static pressure, static temperature and the gas density increases almost instantaneously. Similar results were found in our study. From the study it can be seen that the static temperature and static pressure increase instantaneously downstream the cylinder. The contours of the velocity, pressure, temperature and density showed the results similar to the actual results. Because the shock wave does no work, there is no heat addition. So the total temperature and total enthalpy are constant. But because the flow is non-isentropic, the total pressure downstream the shock wave is less than the total pressure upstream the shock wave. There is a loss of total pressure associated with the shock wave. But a change of static temperature, static pressure and density change was happened.

6. CONCLUSION

In this study, a numerical study of supersonic flow over a circular cylinder at two different temperature(300K and 373K) with a wide range of Mach number(2, 4, 6, 7) was done.

Study of the following things, i.e. variation of static pressure for different surface temperature and different Mach numbers, variation of static temperature for different surface temperature and pressure and comparison of static temperature and pressure for two different surface temperature at a constant Mach number was done. It was found that static pressure and static temperature increases almost instantaneously upstream the cylinder and after the peak value both pressure and temperature decreases and tends to a steady value.

NOMENCLATURE

c_p : Specific heat at constant pressure, $kJ \cdot kg^{-1} \cdot K^{-1}$
P : Pressure, Pa
T : Temperature, K
k : Ratio of specific heat
M : Mach number
α : Speedof sound, ms^{-1}
R : molar gas constant, $J.mol^{-1}.K^{-1}$
μ : Dynamic viscosity, Ns/m^2

REFERNCES

[1] Deng-Pan, W., Yu-Xin, Z., Zhi-Xun, X., Qing-Hua, W. and Zhen-Bing, L., 2012. Flow visualization of supersonic flow over a finite cylinder. *Chinese Physics Letters*, *29*(8), p.084702.

[2] Poplavskaya, T.V., Kirilovskiy, S.V. and Mironov, S.G., 2016, October. Supersonic flow around a cylinder with front gas-permeable insert which modeled by skeleton of porous material. In *AIP Conference Proceedings* (Vol. 1770, No. 1, p. 030067). AIP Publishing.

[3] Bashkin, V.A., Egorov, I.V., Egorova, M.V. and Ivanov, D.V., 2001. Supersonic flow past a circular cylinder with an isothermal surface. *Fluid Dynamics*, *36*(1), pp.147-153.

[4] Rajani, B.N., Kandasamy, A. and Majumdar, S., 2009. Numerical simulation of laminar flow past a circular cylinder. *Applied Mathematical Modelling*, *33*(3), pp.1228-1247.

[5] Fox, R.W. and McDonald, A.T., 1994. Introduction to Fluid Mechanics, John Wiley&Sons. *Inc., New York*.

[6] Anderson Jr, J.D., 2010. *Fundamentals of aerodynamics*. Tata McGraw-Hill Education.

[7] Potter, M.C., Wiggert, D.C. and Ramadan, B.H., 2016. *Mechanics of fluids*. Nelson Education.

[8] Settles, G.S., 2006. High-speed imaging of shock waves, explosions and gunshots: New digital video technology, combined with some classic imaging techniques, reveals shock waves as never before. *American Scientist*, *94*(1), pp.22-31.