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Effect of Inertia and Gravity on Three Dimensional Non-isothermal Film Stability

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ABSTRACT

In three dimensional Newtonian film casting process inertia and gravity plays an effective role on the physical mechanism in non-isothermal condition. This paper investigates the effect of inertia and gravity on critical draw ratio of non-isothermal film casting process using both linear stability analysis. An Eigen value problem is solved from a system of linear ODE equations as nonlinear two-point boundary value problem. Neutral stability curves indicate that in non-isothermal condition gravity and inertia tends the process to be more stable than in isothermal condition. The disturbance frequency is found to be more sensitive to thermal effect. The growth rate of oscillation increases (decreases) above (below) the critical draw ratio results show that the critical draw ratio for the effect of different parameters is greater than the classical value of Drc=20.218 for isothermal condition.

Keywords: Non-isothermal, Film casting, draw resonance, draw ratio, Stability

1. Introduction

In the process of casting polymer melt is extruded through a slit die to form a shape of sheet and stretched by a chill roll after traveling some distance through ambient air. The polymer is then quenched and prepared for further processing or wind up. The distance between the die exit and chilled roller is relatively small. Mainly the film is cooled by the chill roller but there also occurs a solidification process between die exit and chill roll while travelling thorough the ambient air. Generally chill roller take-up velocity is greater than extrusion velocity from die exit. The ratio of take up velocity to die exit velocity is termed as draw ratio (Dr). This is an important parameter in the analysis of film casting stability. As the film is drawn uniaxially, the film thickness is reduced in response to the conservation of mass to a predetermined value at the take-up point by choosing the appropriate draw ratio. Although a larger draw ratio for a higher production rate of polymer is generally expected, this process, however, is limited by an instability known as “draw resonance”. Thus, there exists a critical draw ratio above which periodic variation in film thickness occurs, as observed in practice.

Significant experimental and theoretical efforts had done from the half of last century to study various parameters on the draw resonance instability [1-3]. The mathematical model was first developed by Kase and Matsuo [4,5]. In another investigation Pearson and Matovich [6] analyzed a linear stability for a Newtonian fiber spinning process. At the same time Gelder [7] analyzed the problem neglecting gravity, inertia and surface tension effects for a Newtonian fiber spinning in isothermal condition and Gelded [7] approximate the critical draw ratio is 20.21. However Pearson and Matovich [6] showed in their analyses that for the instability of the system the critical draw ratio is approximately 20.2 for a Newtonian model. In agreement with the experimental study of Donnelly and Weinberger [8]. Using a modified Giesekus rheological model Iyengar and Co [9] investigated the stability of polymeric film casting to infinitesimal and finite amplitude disturbances.

In general sense fiber spinning and film casting are related, there are some basic differences between them. The flow and the disturbances in fiber spinning are considered to be axisymmetric, whereas the flow in film casting is planar and disturbances are generally three dimensional [10]. For the first time draw resonance study in film casting process was performed by Yeow [11] assuming a film of small thickness and infinite width which leads to a one dimensional viscous Newtonian model. Neglecting the effects of inertia and gravity on the Newtonian film casting he investigated stability using the classical hydrodynamic linear theory and the resultant eigen-value problem was numerically solved.

Two dimensional fiber drawing in non-isothermal condition are also studied in the literature [12-17]. Alaie and Papanastasiou [18] studied the film casting of a BKZ-type fluid in an isothermal and non-isothermal steady-state analysis. It was concluded that thinning of the extruded film is enhanced by shear thinning and by air cooling often down to solidification at about the glass transition temperature upstream the chillroll. Regarding Newtonian films. Scheid et al. [19] extended Yeow’s [11] model to account for non-isothermal effects. He analysis the one dimensional model in terms of Stanton (St) number between two region, one is advection dominated cooling for St<<1 and another is transfer dominated cooling for St>>1. Due to the typical geometry of large film widths compared to the thickness, surface tension effects are usually negligible. Extensional study cover the effects of inertia and gravity on the stability behavior in the one dimensional case of infinite width are available from Cao et al. [20] and Bechert et al. [21]. In isothermal condition and for two dimensional film inertia or gravity was found to have

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profound effect on the draw resonance. Ahmed et al. [24] performed a three dimensional linear stability analysis of the film casting process in isothermal condition accounting gravity and inertia effects, they observed a stability enhancement caused by the three dimensional perturbations. Bechert et al. [25] investigated the stability nature of draw resonance under the influence of inertia, neck-in and gravity in isothermal condition. As such it appears that the effect of inertia and gravity on three dimensional non-isothermal film casting has previously not addressed adequately in the past and the current paper will investigate this effect systematically.

2. Mathematical model and Boundary conditions:
In this section, the general formulation is implemented for non-isothermal Newtonian fluid film casting as extension of Ahmed et al. [24]. Consider a non-isothermal and incompressible thin film of Newtonian fluid, is continuously drawing from an extrusion die. The film is taken up by a winder or chill roll placed at a certain distance L from the die. The problem is defined by the Fig.1. Let the density and viscosity of thin-film flow of an incompressible Newtonian fluid are ρ and μ respectively. In this study inertia and gravity are assumed to be relatively important compared to surface tension effect.

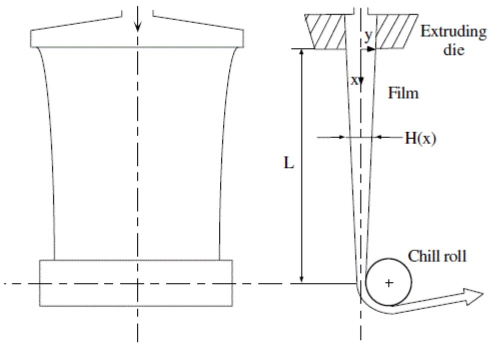

Fig.1: Schematic of the film casting process and non-dimensional coordinates and variables.

The film thickness and velocity are respectively H_0 and U_0 at the exit of Die. The film length between the Die and winder/chill roll is L. The width of the film is very large compared with its thickness and is much larger than L.
Let u, v and w denote the dimensionless velocities in the streamwise, spanwise and depthwise directions respectively and h is dimensionless film thickness. The dimensionless co-ordinates are x, y, z and time t. The aspect ratio will be denoted by $\epsilon = H_0/L$ and will be considered small. u is taken to depend only on x, y and t. Five dimensionless parameters come in the problem Reynolds number Re, Froude number Fr (or equivalently G), Peclet number Pe, Boit number Bi and the draw ratio Dr introduces as,

$Re=\frac{\rho U_0 L}{\mu}$, $Fr=\frac{U_0^2}{gL}$, $G=\frac{Re}{Fr}=\frac{\rho g L^2}{\mu U_0}$, $Dr=\frac{U_L}{U_0}$, $Pe=\frac{\rho c_p U_0 L}{k}$ and $Bi=\frac{h_m L^2}{k H_0}$

where h_m convective heat transfer co-efficient and U_L is the velocity at the chill roll. Ahmed et al. [24] derived the continuity and momentum conservation equations based on Schultz et al. [13] asymptotic analysis which is the base of our paper. This paper used the final deduced form of continuity and momentum conservation equations of Ahmed et al. [24] except further deduced the procedure.

$$\delta_t + (\delta u)_x + (\delta v)_y = 0$$

$$Re\,\delta(u_t + uu_x + vu_y) = [\delta(4u_x + 2v_y)]_x + [\delta(u_y + v_x)]_y + \delta G$$

$$Re\,\delta(v_t + uv_x + vv_y) = [\delta(4v_y + 2u_x)]_y + [\delta(u_y + v_x)]_x \quad (1)$$

The conservation of energy equation becomes (neglecting viscous dissipation)

$$\rho c_p \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = k\left(\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y} + \frac{\partial^2 T}{\partial^2 z}\right) \quad (2)$$

Apply kinematic condition at free surface boundary value in the energy equation and integrating with respect to z over h_1 to h_2 (let $\delta = h_2 - h_1$)

$$\rho\, c_p\, \delta \left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) = \delta k\left(\frac{\partial^2 T}{\partial^2 x} + \frac{\partial^2 T}{\partial^2 y}\right) - 2k\,[h_m(T - T_s)] \quad (3)$$

Simplifying by dimensionless co-ordinates and numbers the equation becomes

$$Pe\left(\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y}\right) = \frac{\partial^2 \theta}{\partial^2 x} + \frac{\partial^2 \theta}{\partial^2 y} - \frac{2Bi\theta}{\delta} \quad (4)$$

3.0 Steady State solution
Here we analysis the energy equation to study non-isothermal condition only. Governing equation of steady state are considered as follows

$$Re\,\delta_s u_s u_s' = 4(\delta_s u_s'' + \delta_s' u_s') + \delta_s G$$
$$(\delta_s u_s' + \delta_s' u_s) = 0$$
$$Pe\, u_s \theta_s' = \theta_s'' - 2Bi\theta_s/\delta \quad (5)$$

Where the subscript s indicates the steady state and a prime denotes total differentiation with respect to x. The corresponding boundary conditions are given by as follows:

$u_s(x=0)=1$ $u_s(x=1)=Dr$ $\delta_s(x=0)=1$ $\theta_s(x=0)=1$,
$\theta_s'(x=1)=0$

In isothermal condition including gravity and inertia fang et.al [21] experimented the flow in the range of inertia Re ∈[0,0.2] and gravity G ∈[0,100]. Based on the rheological properties of polymer fluid the ranges are chosen considering commercial film casting and fiber spinning process.

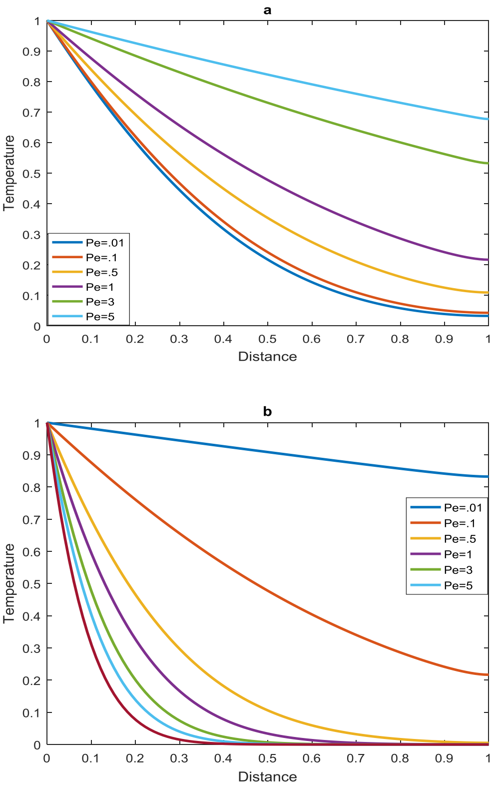

Fig.2: Influence of Peclet number (a) and Biot number (b) on the steady state temperature distributions at G=50, Re =0.01, DR = 20.0, Bi=1(a) and Pe=1(b).

In our study we take this range for the ease of compare-son with previous result to better illustrate the non-isothermal effect. Our results agreement closely with Fang et al. [10] for velocity and thickness near the die exit and take up point. Stretching velocity is higher at take up point at inertia increases. Gravitational effect is low at G<15. As the high temperature liquid film travel through air gap there occurs heat transfer to ambient air. Bulk fluid are stretched by chill roller from the die exit and heat diffusion occurs travelling through air and the heat transfer occurred through surface. Fig.2 shows that (a) as the increase of Peclet number the temperature is dissipated more slowly with the distance from die exit and Fig.2 (b) as the increase of Biot number the tempe-rature is dissipated more rapidly with the distance from the die exit. In polymer film casting Peclet number is high and Biot number is moderately low in practical cases.

4.0 Linear Stability Analysis

Solving the eigenvalue problem neutral stability curve are obtained and resulting draw ratio is called critical draw ratio. The imposed infinitesimal perturbation amp-lify exponentially with time above this critical value considered infinitesimal disturbances as super position to steady state flow. Thus perturbed steady state tempe-rature film thickness and velocities are as

$$\begin{aligned} u(x,y,t) &= u_s(x) + U(x)\, e^{\lambda t+iky} \\ v(x,y,t) &= V(x)\, e^{\lambda t+iky} \\ \delta(x,y,t) &= \delta_s(x) + \Delta(x)\, e^{\lambda t+iky} \\ \theta(x,y,t) &= \theta_s(x) + \Theta(x)\, e^{\lambda t+iky} \end{aligned} \qquad (6)$$

where δ_s and u_s indicate the steady state solutions and Δ, U, and V are complex perturbation amplitudes. k repre-sents the real wavenumber for the disturbances along the span wise direction. In this case, k is the complex eigen-value (with λ_r being the growth or λ_i decay rate and being the disturbance frequency). Assume the flow velocity in the depth wise direction (z) is decoupled from the flow in the(x, y) plane and is dictated by the continuity equation. Upon the substitution of expres-sions (6) into Eqs. (1), (4) and eliminating δ_s, δ_s', u_s'' in terms of u_s and u_s' using steady state equation the linearized Eqs. are rearranged and the disturbance fre-quency is recast as a first order equation to yield the perturbation variables are found to satisfy the following linearized differential equations are rearranged

$$\begin{aligned} U'' &= (\mathrm{Re}u_s/4+2u_s'/u_s)\, U' + (\mathrm{Re}u_s'/4-u_s'^2/u_s^2 + k^2/4)\, U \\ &\quad +\lambda\, (\mathrm{Re}U/4+\Delta u_s') + 3iku_s'V/2u_s - 3ikV'/4 \\ V'' &= (\mathrm{Re}u_s/4+u_s'/u_s)\, V' + 4k^2 V \\ &\quad +\lambda \mathrm{Re}V - 3ikU' + (iku_s'/u_s)\, U \end{aligned} \qquad (7)$$

$$\Delta' = (u_s'/u_s^3)U - \lambda\Delta/u_s - U'/u_s^2 - u_s'\Delta/u_s - ikV/u_s^2$$

$$\begin{aligned} \Theta'' &= \mathrm{Pe}(\lambda\Theta + u_s\Theta' + \theta' U + u_s\theta'\Theta/\delta_s) + k^2\Theta + 2\mathrm{Bi}\Theta/\delta_s - \\ &\quad (\Delta/\delta_s)\, (\mathrm{Pe}\, u_s\theta_s' + 2\mathrm{Bi}\theta_s/\delta_s) \end{aligned}$$

the corresponding boundary conditions are

$U = V = \Delta = \Theta = 0$ and $U' = 0$ at x=0,
$U = V = \Delta = \Theta = 0$ at x=1

Ahmed et al. [24] is solved the linear eigen-value prob-lem as a nonlinear two-point boundary value problem. Only the disturbance frequency is considered unknown variable in this case while the critical draw ratio is determined upon using an iterative technique. λ'=0. where a prime denotes total differentiation with respect to the problem is solved by MATLAB using the function 'bvp4c'. The collocation polynomial provides a fourth-order accurate C1-continuous solution in the interval of integration.

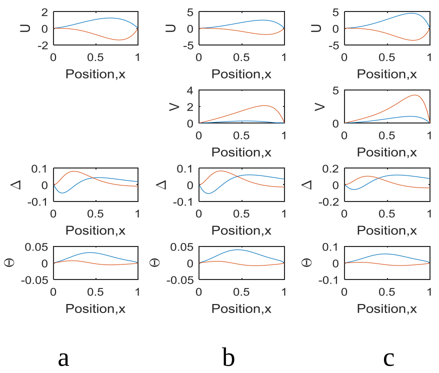

Fig.3: Solutions to the eigenvalue problem at Dr=25 for G=10, Pe=1, Bi=5 (a) k=0, (b) k=2, and (c) k=4, with the blue line and the red line representing the real and imaginary parts of complex-valued perturbations respectively.

The Fig.3 indicates the flow strengthening resulting from three dimensional. Overall amplitudes of U, Δ and Θ are not influenced significantly. The extreme position of U are changed with the increasing of wavenumber k, real part of V remain close to zero whereas Δ and Θ are not effectively influenced.

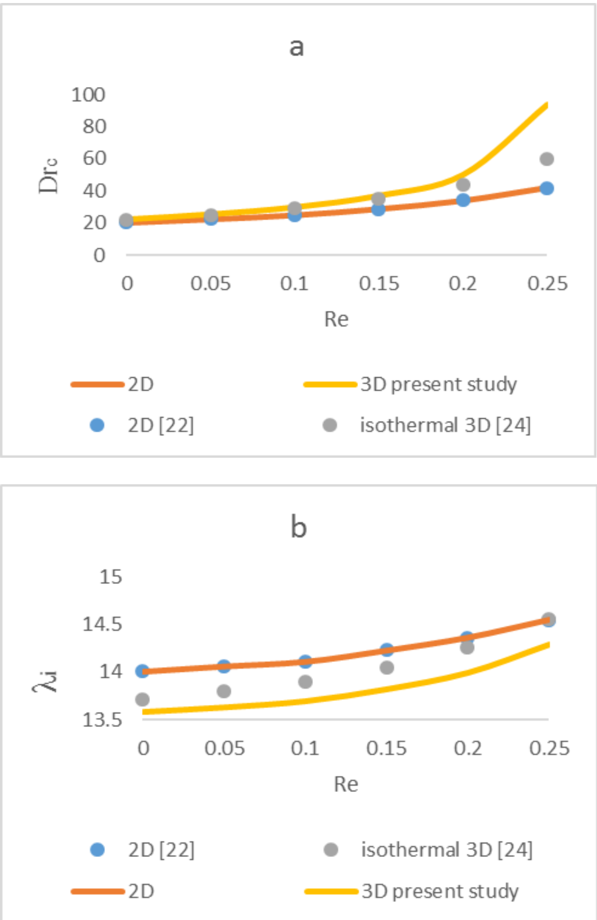

Fig.4: Comparison of (a) critical draw ratio and (b) disturbance frequency with 2D [22] and isothermal 3D [24] disturbance by taking G=0, Pe=0, Bi=0.

Note that the 2D results from the current formulation (by setting k=0) almost overlap with those obtained by Fang et al. [10] and non- isothermal 3D results from current formulation (by setting k=1, Pe=0, Bi=0) in the absence of inertia and gravity is predicted as 22.637 which is higher than the isothermal 3D critical value of 22.15 obtained by Ahmed et al. [24].
The comparison of neutral stability curves for 2D, isothermal 3D and non-isothermal 3D flows is depicted in Fig. 4 That gives an indication that the non-isothermal 3D film flow is more stable, at least in the absence of inertia and gravity.

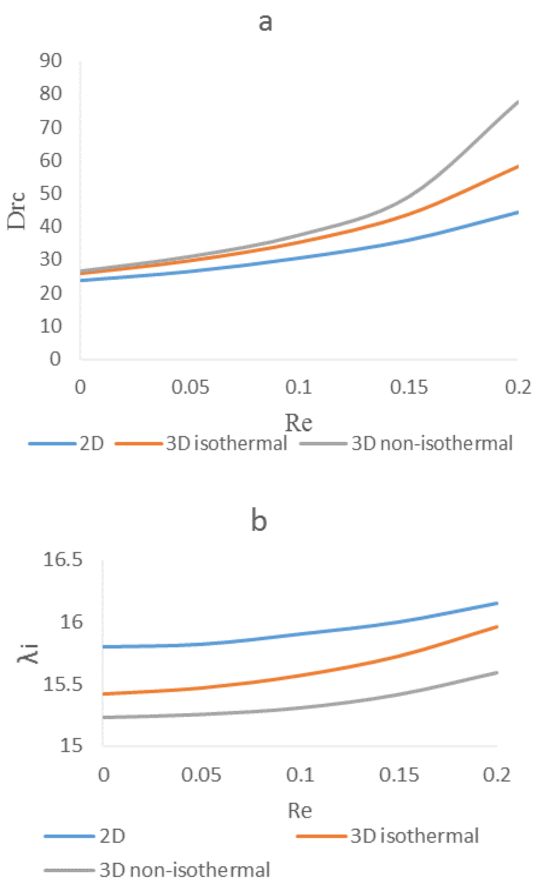

Fig.5: comparison of (a) the critical draw ratio and (b) disturbance frequency for 2D, isothermal and non-isothermal (Pe=1, Bi=1) 3D flows for G=10.

The enhanced stability is even more evident at higher inertia, as shown in Fig.4. In contrast, the oscillation frequency is lower for the non-isothermal 3D flow. At wide range of inertia. However, at large inertia (Re>0.2) the frequency for isothermal 3D flow is higher than the 2D flow but the frequency for non-isothermal 3D flow is relatively lower at large inertia. This is shown in Fig. 4. Both inertia and gravity have a significant stabilizing influence on the film casting in non-isothermal condition. This is illustrated in Fig.5, where the critical draw ratio is plotted against Re for G=10. The critical draw ratio increases with Re, indicating that inertia delays draw resonance. The non-isothermal effect is more influential at higher inertia say, Re >0.2, as shown in Fig.5. The discrepancy between the predictions based on linear stability and reality can then be explained by the stabilizing influence of inertia and gravity in non-isothermal condition. The disturbance frequency is increased monotonically with both Re and G. Since the typical draw ratio in an industrial film casting process is about 28 to 31 for polypropylene and somewhat higher for polyethylene [13], it is assumed that for higher inertia (say, Re>0.20) the film casting process is stable for all practical purposes.

Here the critical draw ratio is higher for non-isothermal 3D flow than isothermal 3D flow. On the contrary, the disturbance frequency is lower for non-isothermal 3D flow. In Fig.6 shows the increase in wavenumber with inertia indicates an almost complete stabilization of the practical processes. The disturbance frequency decrease with the increase of wavenumber**.**

5.0Transient response of inertia and gravity in non-isothermal flow

Linear stability analysis is useful only in predicting the onset conditions of the instability in film casting, but fails to predict the actual spatio-temporal response once analysis is conducted here to examine the stability of the system to finite amplitude disturbances. The nonlinear response in the absence of inertia and gravity examined by Iyengar and Co [11] using a finite-element approach to discretize the full equations spatially and resulting equations are integrated in time. In the present analysis, the system of nonlinear time-dependent partial differential equations is discretized using a finite-difference method and solved as a boundary value problem.

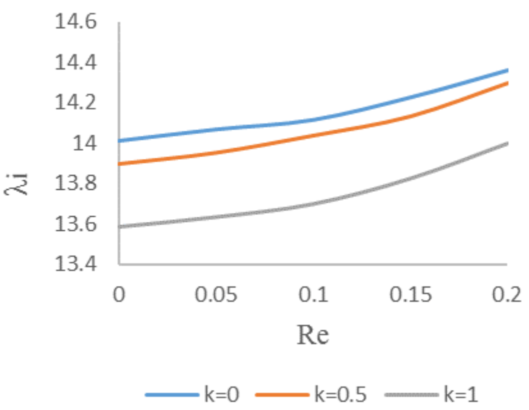

Fig.6: Influence of wavenumber on the disturbance frequency over the range Re ϵ [0, 0.2], k ϵ [0, 1], Pe=1 and Bi=1.

The disturbances are introduced by adding finite amplitude disturbances onto the steady-state solutions. Long time is needed (1–10 h of CPU time) per run as small time step ($\Delta t = 0.01$) needed for convergence. To obtain an accurate numerical result, an extremely small time step is used, i.e. $\Delta t = 0.001$. The nonlinear analysis is conducted for several draw ratios corresponding to draw ratios below, near and above the critical value. The response of the film thickness at the take-up point ($x = 1$) is displayed in Fig.8 Linear stability analysis predicts the onset of a Hopf-bifurcation (over stability) at the critical draw ratio.

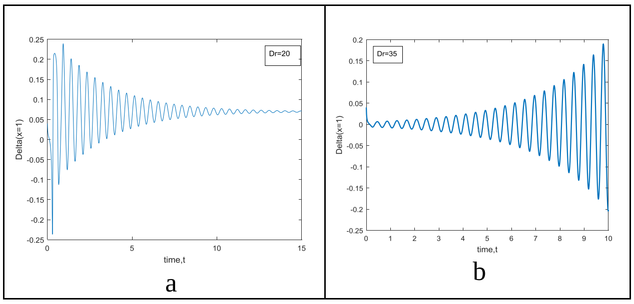

Fig.7: Transient response of film thickness at take-up point h ($x = 1$, t) for draw ratio (a) 20 and (b) 35 for Re=.1, Pe=474 and Bi=750.

The Fig.7 shows that for a draw ratio of 20.0 the film thickness at the take-up point exhibits damped oscillation. The disturbance decays with time, but remains symmetric with respect to the steady state. For a draw ratio of 35.0, the disturbance is amplified and grows relatively slowly. Due to the slow growth of the amplitude, sustained oscillation will not be reached until a time large enough. It is noted that beyond the critical draw ratio the sustained disturbance consists of narrow, sharp peaks alternating with wide, flat bottoms, reflecting a dissymmetry with respect to the steady state.

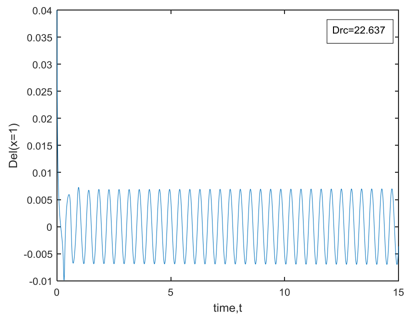

Fig.8: Transient response of film thickness for Re=0, G=0, Pe =1 and Bi= 1.

It can be seen that increasing of Biot number as well as decreasing of peclet number improves the flow stability as Tian et al. [23] investigated the effect of cooling on the stability of film casting with three different convective heat transfer coefficients (h_m). He found the increase in h_m reduces the amplitude of the draw resonance significantly.

6.0 Conclusion

In the non-isothermal film casting process the role of inertia and gravity in draw resonance is investigated for a Newtonian film both linear and nonlinear stability analysis, which focuses on the transient response are

carried out in this study. For linear stability analysis, the eigenvalue problem is treated as a two-point boundary-value problem including the steady state which is determined simultaneously.
The steady state film thickness (velocity) is found to increase (decrease) with the increasing inertia. Gravitational effect has the opposite influence. Also Peclet number and Biot number has opposite influence. Increasing of Biot number improves the flow stability. It is concluded here that although both inertia and gravity stabilize the process, inertia is more influential than gravity on the stability. Certain combinations of the flow parameters, lead to a dramatic increase in the critical draw ratio by non-linear analysis.

NOMENCLATURE

Dr Draw ratio
U_o Velocity
H_o, δ Thickness
μ Viscosity
ρ Density
Drc Critical draw ratio

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