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## Numerical Investigation of Laminar Convective Heat Transfer and Friction Factor of a Pipe by Using Al<sub>2</sub>O<sub>3</sub>-Water Nanofluid

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### ABSTRACT

The Numerical study of laminar convective heat transfer of aluminum oxide (Al<sub>2</sub>O<sub>3</sub>) - water nanofluid for the developed region through a plain tube is presented. The second order single phase energy equation, mass and momentum equation are solved by using finite volume method with the ANSYS FLUENT 16 software. The plain pipe's diameter is 5mm and length is 750mm. Aluminum oxide (Al<sub>2</sub>O<sub>3</sub>) nanoparticles with different volume fraction (1% - 5%) using with water which is considered as the base fluid are analyzed for a range of Reynolds number from 100 to 1400 at constant heat flux 500 W/m<sup>2</sup> at the tube wall. The result reveals that for increasing the Reynolds number the Nusselt number and heat transfer coefficient are increased linearly and friction factor decreased linearly in the developed region for both water and Al<sub>2</sub>O<sub>3</sub>-H<sub>2</sub>O nanofluid. At constant Reynolds number, by increasing the volume fraction of Al<sub>2</sub>O<sub>3</sub> nanoparticles from 1% to 5% the value of Nusselt number increased rapidly from 0.27% to 15%, heat transfer coefficient increased 7.2% to 31.5% and friction factor increased very little from 0.1% to 2%.

Keywords: Convective heat transfer, friction factor, nanofluids, nanoparticles volume concentration and thermal conductivity.

### 1. Introduction

The most fundamental and effective topics of thermal engineering is heat transfer which influences our everyday life significantly and discusses about generation and creation, demeanor, principle, metamorphosis and permutation of thermal energy and heat between physical systems. This is the main stagnation of thermodynamics. Effective amount of heat removal or addition moving from one process stream to another is one of the major challenges in numerous industries, including power generation, transportation, manufacturing, air conditioning, cooling, heating, lubricating etc. Apart from conduction fluid also carry out energy and heat. In automobiles, refrigerators, heat exchangers, air-condition system, cooling and lubricating system etc. system's energy transfer is carried by fluid. On the other hand fluid is the main phenomena to control the behavior of heat transfer. From last few decades to improve heat transfer efficiency, heat transfer rate, thermal conductivity and to reduce pumping power, pressure loss, frictional loss between any physical systems several methods has been developed. Among these methods some are utilization of extended surface of the physical system, application of vibration to the heat transfer surface, use of micro channels, utilization of fins etc. But one important investigation is heat transfer can also be improved significantly by increasing the thermal conductivity of the working fluid. And so at present maximum researchers give more concentration on improving the thermal conductivity of working fluids because water, ethylene glycol and engine oil which are commonly used as heat transfer fluid but this fluid have relatively low thermal conductivity when compared to the thermal conductivity of solids. So by adding small amount of solid particles with the base or working fluid, the thermal conductivity of

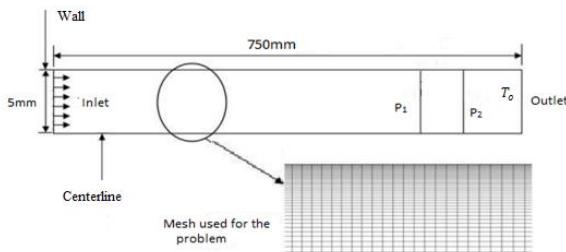
the fluid can be increased noticeably. And by using these concept researchers has been made nanofluid which is the combination of base fluid (water, engine oil or ethylene glycol) and very small amount of solid particles at Nano scale size (1nm to100nm). Al<sub>2</sub>O<sub>3</sub>, CuO, TiO<sub>2</sub>, SiC, SiO<sub>2</sub>, Fe<sub>2</sub>O<sub>3</sub>, MgO etc. particles are used as nanoparticles to mix with base fluids. Different researchers carried out their investigation on nanofluids. In 1995-1996 Choi and Eastman [1] reinvestigated with their Nano scale metallic particle and carbon nanotube suspensions in Argonne National Laboratory. Choi and Eastman have tried to suspend various metal and metal oxides nanoparticles in several different fluids (Choi (1998) [2]; Choi et al.(2001)[3]; Choi et al. (2005); Choi et al. (2006); Eastman et al. (2001)[4]; Eastman et al. (1999); Eastman et al. (2004)) and their result are promising. From their investigation they also observed that many things remain vacuous about these suspensions of Nano-structured materials, Choi and Eastman have been termed these as "Nano fluid" Xuan and Li, 2003[5] examined the convective heat exchange and the stream highlights of Cu- water Nano fluids in a 10-mm inward distance across tube. The trial comes about because of their investigation, in the turbulent area showed that the friction factors of the Nano fluids, between 1 and 2 vol. % fractions are generally the same as those of water flow. Williams et al., 2008 [6] experimentally explored the turbulent stream of alumina- water and zirconia- water Nano fluids in tubes. They found that current connections for single-stage stream can enough anticipate Nano fluid stream convective heat exchange and weight drop. Rea et al., 2009[7] led an investigation on the laminar convective warmth exchange and weight drop of alumina- water and zirconia- water Nano fluids in a tube with 4.5-mm inward width. Their discoveries demonstrated that, with

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appropriately measured Nano fluid properties, there is no deviation in convective heat exchange and weight drop of Nano fluid spill out of customary single-stage stream hypothesis. Heris et al.,2013 [8] played out an exploratory investigation to decide the pressure drop and heat exchange qualities of Al<sub>2</sub>O<sub>3</sub>/water and CuO/water nanofluids in a triangular conduit under consistent heat flux where the flow was laminar. Their outcomes demonstrated that, at similar estimations of nanoparticle volume division and Reynolds number, utilizing Al<sub>2</sub>O<sub>3</sub> nanoparticles is more beneficial than CuO nanoparticles. Wen and Ding, 2004[9] also investigate with laminar convective heat transfer using Al<sub>2</sub>O<sub>3</sub> nanofluid. Hwang and Choi, 2009[10] also worked on it and showed that 3% volume concentration Al<sub>2</sub>O<sub>3</sub> nanofluid gives 8% enhancement of heat transfer coefficient N.K Chavda and Makwana [11] carried an experiment on different pipe and pipe fittings to investigate the friction factor and loss coefficient by using CuO/water nanofluid and the result shows that friction factor increases with the increment of concentration of CuO/water nanofluid compared to water.

## 2. Physical Model and Boundary Conditions:

A two dimensional tube with a steady heat flux that is supplied on its surface to investigate heat transfer rate and corresponding friction factor. In order to investigate the performance of Nano fluid through a tube, a numerical study has been carried out by employing the well-known commercial computational fluid dynamics software ANSYS Fluent. A laminar flow through a two dimensional circular shape pipe with 5mm diameter and 750 mm length is presented. A constant uniform heat flux of 500 W/m<sup>2</sup> is applied at the wall boundary of the tube and fluid is permitted to stream with a fitting speed and uniform temperature of 303 K at the inlet of the tube with a presumption of no slip condition on the tube wall. All the fluid dynamic and heat exchange parameters are extricated after the hydrodynamic and thermal improvement of the fluid stream and in this case the entrance length is  $x/D=60$  beyond which all the measurements are taken. For calculating the heat transfer enhancement and friction factor the temperatures are taken at a line which is situated 750mm from inlet and pressures are taken at lines 725mm and 715mm from the inlet.



**Fig.1** Physical model of the numerical problem and the corresponding mesh of the domain (Tube)

## 3. Numerical Method and Methodology:

We use a commercial computational fluid dynamics software ANSYS (Fluent) for this numerical analysis. All

the governing equations for mass, momentum, energy, and laminar quantities are solved using a control volume technique. At inlet laminar inlet velocity and at the outlet boundary pressure outlet is considered. Under relaxation factors 0.4 for pressure, 0.785 for momentum equation, 1 for energy equation, and 0.8 for density equation are considered for circular tube. Al<sub>2</sub>O<sub>3</sub>-water nanofluids with different particle volume fractions (1, 2, 3, 4, and 5%) are tested with a wide range of Reynolds number (100-1000 for circular tube) and then results are compared with base fluid water.

### 3.1 Governing Equation:

The governing equation continuity, momentum and energy for forced convection under laminar flow and steady-state conditions are expressed as follows:

Continuity Equation:

In steady flow, the amount of mass within the control volume under remains constant, and thus the conversation of mass can be expressed as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1)$$

Momentum Equation:

For laminar flow, the momentum equation can be expressed as:

$$\rho \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu \frac{\partial^2 u}{\partial y^2} \quad (2)$$

Energy Equation:

Energy can be transferred by heat, work, and mass only, the energy balance for a steady-flow control volume can be write explicitly as

$$\begin{aligned} (E_{in} - E_{out})_{by\ mass} &= -\rho c_p \left( u \frac{\partial T}{\partial x} + T \frac{\partial u}{\partial x} \right) dx \cdot dy - \rho c_p \left( v \frac{\partial T}{\partial y} + T \frac{\partial v}{\partial y} \right) dx \cdot dy \\ &= -\rho c_p \left( u \frac{\partial T}{\partial x} + T \frac{\partial T}{\partial y} \right) dx \cdot dy \end{aligned} \quad (3)$$

### 3.2. Thermal and fluid dynamic Properties of Nanofluids:

The Reynolds number for the flow of Nano fluid is

$$\text{expressed as: } Re = \frac{\rho_{nf} U_{av} D_h}{\mu_{nf}} \quad (4)$$

The rate of heat transfer  $Q_{nf}$  to the tube wall is assumed to be totally dissipated to Nanofluid flowing through a circular tube, raising its temperature from inlet fluid bulk temperature  $T_{bi}$  to exit fluid bulk temperature  $T_{bo}$ . Thus,

$$Q_{nf} = m_{nf} C_{p_{nf}} (T_{bo} - T_{bi}) \quad (5)$$

Where  $m_{nf}$  the mass flow rate of nanofluid is,  $C_{p_{nf}}$  is the specific heat of Nano fluid at constant pressure.

The average heat transfer coefficient  $h_c$  is given by:

$$h_c = \frac{Q_{nf}}{A_w (\Delta T)} \quad (6)$$

Where  $A_w$  is the surface area of circular tube and the temperature difference between the wall and calculated as:

$$T = \frac{(T_w - T_o) - (T_w - T_i)}{\ln\left(\frac{T_w - T_o}{T_w - T_i}\right)} \quad (7)$$

So the expression of average Nusselt number is defined as follows:

$$Nu = \frac{h_c D_h}{K_{nf}} \quad (8)$$

$$\text{Pressure difference: } \Delta P = \frac{f L \rho U^2}{2 D_h} \quad (9)$$

Then, the Darcy friction factor, for laminar flow is:

$$f = \frac{64}{Re} \quad (10)$$

Dynamic Viscosity:

The dynamic viscosity For  $Al_2O_3$ -water Nano fluid is given by Maiga et al.(2004)[12]:

$$\mu_{nf} = (1 + 7.3\phi + 123\phi^2) \quad (11)$$

Thermal Conductivity:

A wide range of experimental and theoretical studies were conducted in the literature to model thermal conductivity of nanofluids. The existing results were generally based on the definition of the effective thermal conductivity of a two-component mixture. . There are several thermal conductivity equations among them we use Pak and Cho equation for  $Al_2O_3$ -water nanofluid. The following formulas are given by Pak and Cho (1998) [13], equation

$$K_{nf} = K_{bf}(1.0021 + 7.3349\phi) \quad (12)$$

Density:

Using classical formulas derived for a two-phase mixture density (Xuan and Roetzel, 2000) of the nanofluid as a function of the particle volume concentration and individual properties can be computed using following equation [14]:

$$\rho_{nf} = \rho_p \phi + \rho_{bf}(1 - \phi) \quad (13)$$

Specific Heat:

Using classical formulas derived for a two-phase mixture, the specific heat capacity (Pak and Cho, 1998) of the nanofluid as a function of the particle volume concentration and individual properties can be computed using following equation [13]:

$$C_{nf} = (1 - \phi)C_w + \phi C_p \quad (14)$$

#### 4. Code Validation Test

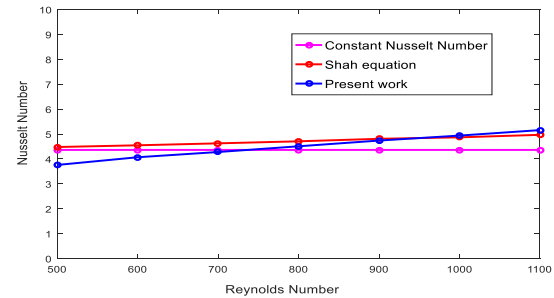
For laminar tube flow at uniform velocity and constant heat flux water has been passed through the tube and a range of Reynolds number of 500-1400 has been considered for calculating Nusselt numbers and a range of

Reynolds number 100-1000 has been considerate for calculating friction factor. At fully developed zone the obtaining Nusselt number is compared with the constant value of Nusselt number for laminar tube flow 4.346 at constant heat flux and the Nusselt numbers obtained from Shah theoretical equation (2009) which is shown in Figure 2 that gives a good agreement with only 2% error and are constant. The friction factor obtaining at fully developed zone is compared with Darcy-Weisbach equation [15] which is shown in Figure 3 that give only 1.5% error. The correlation developed by Shah Equation [15] for laminar tube as follows:

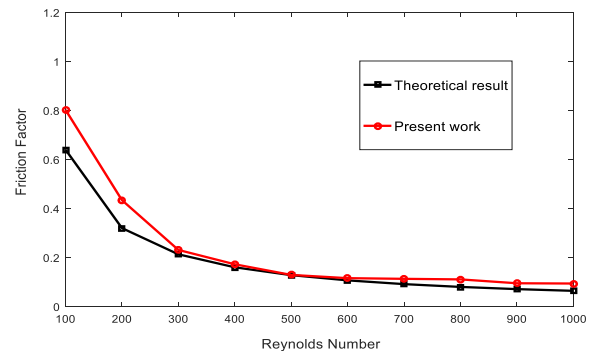
$$Nu = 1.302 \left(\frac{x^*}{2}\right)^{-\left(\frac{1}{3}\right)} - 0.5, \quad x^* \leq 0.003 \quad (15)$$

$$Nu = 4.364 + 0.263 \left(\frac{x^*}{2}\right)^{-0.506} e^{-41\left(\frac{x^*}{2}\right)}, \quad x^* > 0.03 \quad (16)$$

$$\text{Where, } x^* = \frac{2(x/D)}{Re Pr}$$



**Fig. 2:** Comparison of Nusselt number between Shah equation and constant Nusselt number at constant heat flux for laminar tube flow and present work for different Reynolds number of water.

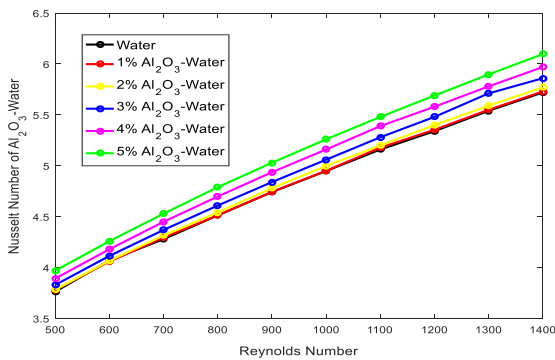


**Fig.3:** Comparison of friction factor between Darcy-Weisbach equation and present work for different Re.

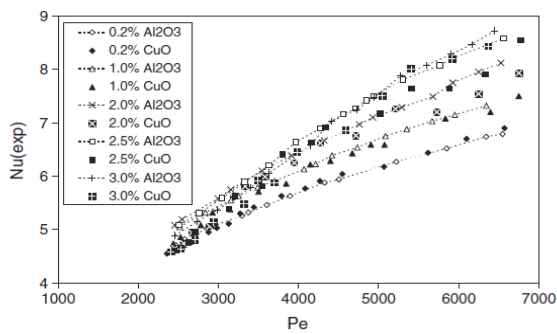
#### 5. Result and discussion:

The figure 4, show the effect of volume fraction and Reynolds number on the Nusselt number for  $Al_2O_3$ -water, from the figure it has been obtained that the Nusselt number of the nanofluid used in the present work increases with the increase in Reynolds number and Volume fraction of the nanofluids. This is occurred because of the increment of the effective thermal

conductivity and an increase of energy exchange rate resulting from the irregular and chaotic motion of ultrafine particles of the nanofluids with the increases of the volume fraction as well as increment of convection with the increase in Reynolds number. This phenomena is explained by Hsien-Hung Ting, 2015 [16]“Actually a higher Reynolds number corresponds to higher fluid velocity and temperature gradient, which in turn results in a higher value of nusselt number”. S. Zeinali Heris, 2006 [8] worked with laminar convective heat transfer of circular tube by using  $Al_2O_3$ -water and  $CuO$ -water nanofluids and investigated that the nusselt number is increased for both nanofluids respectively with increasing the volume fraction of the nanofluids and with the increment of Peclet number that indicates the increment of Reynolds number and this trend is observed in figure 5. Pak and Cho et el [15] also investigated that the Nusselt number for fully developed turbulent flow increased correspondingly to the increasing volume fraction as well as Reynolds number for  $\gamma-Al_2O_3$ -water and  $TiO_2$ -water nanofluids which trend is also similar to the present work.



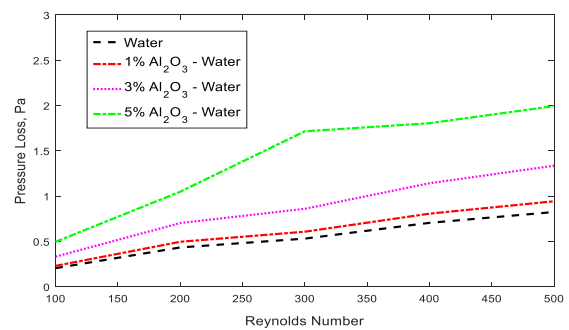
**Fig. 4:** Comparison of Nusselt Number of different volume fraction  $Al_2O_3$ - $H_2O$  for different Reynolds Number



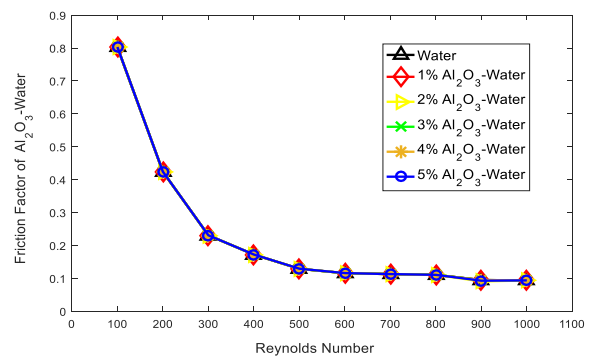
**Fig. 5:** Experimental Nusselt number versus Peclet number nanofluids (S. Zeinali Heris, 2006 [8])

Fig. 6 indicates the pressure drop of nanofluid with increase of Reynolds number at different volume fractions. From the fig it is observed that for different nanofluid pressure drop is increasing with increase of Reynolds number for different volume fractions and the pressure drop is becoming higher with increase of volume fraction. This increment is comparatively more than pure water. The figure 7 shows the comparison of friction

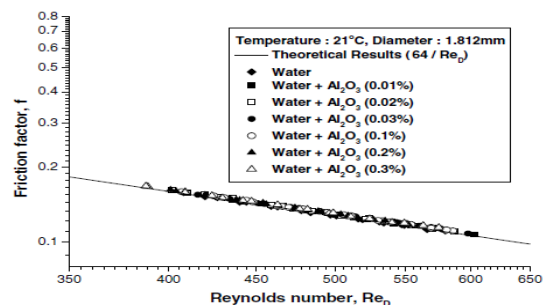
factor of different nanofluids with pure water for different Reynolds number. From figure it has been observed that the friction factor of the four nanofluids is remaining almost same. And by increasing the volume fraction of nanoparticles the friction factor increases very slightly 0.25% to 0.85% than the pure water. This similar trend is also reported by Hwang and Choi, 2009[16] (showed at figure 8) with  $Al_2O_3$ -water under laminar tube flow and Xuan and Li, 2000 and Pak and Cho, 1998 [10] under turbulent flow. According to Li and Xuan, 2000[14] the friction factor of  $Cu$ -water nanofluid with low volume fraction of particles is almost not changed that satisfied the present result.



**Fig. 6:** Comparison of pressure loss with Reynolds number for different Volume Fraction of  $Al_2O_3$ -water.



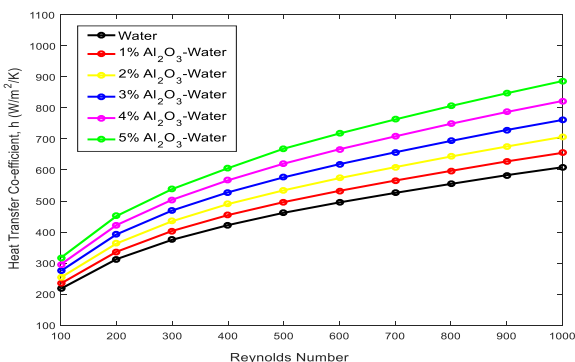
**Fig. 7:** Comparison of friction factor and Reynolds number of different volume fraction of  $Al_2O_3$ -water.



**Fig. 8:** The friction factor of water-based  $Al_2O_3$  NF in fully developed laminar flow. (Hwang, 2009[10])

The figure 9 represents the effect of volume fraction of nanofluids on heat transfer co-efficient for different

Reynolds number. From the figures it is observed that the value of heat transfer coefficient increases rapidly with the increase of volume concentration and Reynolds number. This is due to the increases of the thermal conductivity and decreases of the specific heat capacity of the Nano fluid which increases the Nusselt number with higher velocity and temperature gradient and this phenomenon increases the heat transfer coefficient gradually. According to Wen and Ding, 2006[9] the numerical study of the particles migration due to viscosity gradient and a non-uniform shear rates leads to higher heat transfer coefficient of nanofluids. From the figures the increment of heat transfer coefficient compared to pure water is 7.2% to 31.5% for Al<sub>2</sub>O<sub>3</sub>-water with respect to 1% to 5% volume concentration for Reynolds number 500 to 1400. S. Zeinali Heris and Esfahany, 2006[8] investigated convective heat transfer of circular tube by using Al<sub>2</sub>O<sub>3</sub>-water nanofluid and observed that the heat transfer coefficient increases rapidly with the increases of Reynolds number with respect to increase of volume concentration that satisfied the present work. Another same investigation was carried by Lin-wen Hu , 2000[18] with Al<sub>2</sub>O<sub>3</sub>-water and Zirconia-water and reported that at fully developed region the heat transfer coefficient increases 27% for Al<sub>2</sub>O<sub>3</sub>-water and 3% for zirconia-water nanofluid that also almost similar to the present work. Sezer Ozerinc, 2010 [19] also proposed that for Al<sub>2</sub>O<sub>3</sub>-water nanofluid the heat transfer coefficient increases rapidly compare to pure water with the increment of Reynolds number and volume fraction.



**Fig. 9:** Comparison of Heat Transfer Coefficient with Reynolds of different volume fraction of Al<sub>2</sub>O<sub>3</sub>-water.

The table 5.1 shows the performance comparison of different volume fraction of Al<sub>2</sub>O<sub>3</sub>-water nanofluid with base fluid. From the table it is clear that at constant heat transfer coefficient the Reynolds number, Nusselt number and velocity of nanofluid has been reduced and the thermo physical property likes thermal conductivity, density, viscosity has been increase by increasing the volume concentration compared to pure water. . And to get same heat transfer coefficient nanofluid needs lower volumetric flow rate compared to pure water. This reduction is 11.25% for 1% Al<sub>2</sub>O<sub>3</sub>-water and maximum 40.39% for 5% Al<sub>2</sub>O<sub>3</sub>-water nanofluid compared to pure water. And to get this advantage of volumetric flow rate and other fluid parameters the friction factor of Al<sub>2</sub>O<sub>3</sub>-water

nanofluid is increased very small amount compared to pure water.

**Table 1:** Comparison of the performance of different volume concentration of Al<sub>2</sub>O<sub>3</sub> nanoparticles with base fluid.

Type of fluid parameters	Water	1% Al <sub>2</sub> O <sub>3</sub>	2% Al <sub>2</sub> O <sub>3</sub>	3% Al <sub>2</sub> O <sub>3</sub>	4% Al <sub>2</sub> O <sub>3</sub>	5% Al <sub>2</sub> O <sub>3</sub>
Heat Transfer coefficient W/m <sup>2</sup> .K	600	600	600	600	600	600
Reynolds number	960	810	675	550	463	395
Nusselt number	4.85	4.55	4.25	4.00	3.8	3.65
Velocity m/s	0.1538	0.13677	0.121995	0.10744	0.098666	0.091788
Friction factor	0.0955	0.1122768	0.11779643	0.13566956	0.19978800	0.2864115
Thermal conductivity	0.615	0.6614	0.7065	0.7516	0.79678	0.8418
Density Kg/m <sup>3</sup>	996	1025.74	1055.48	1085.22	1114.96	1144.7
Volumetric flow rate, m <sup>3</sup> /s	3.015e-6	2.68e-6	2.395e-6	2.105e-6	1.934e-6	1.799e-6
Reduction in volumetric flow rate	-	11.25%	20.5%	30.46%	36.092%	40.39%

## 8. Conclusion

In the present work  $\text{Al}_2\text{O}_3$ -water nanofluid have been analysed through a typical circular tube to investigate the heat transfer enhancement and the friction factor. The heat transfer coefficient, Nusselt Number and Pressure drop increase with the increase in volume fraction for the nanofluid as well as with the Reynolds number compared to pure water. And to get more heat transfer coefficient and volumetric flow rate advantage use of  $\text{Al}_2\text{O}_3$ -water nanofluid is best compared to pure water.

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## Nomenclature

- $C_p$ : Specific heat at constant pressure  
 $D_h$ : Hydraulic diameter  
 $f$ : Friction factor  
 $h$ : Average heat transfer coefficient  
 $k$ : Thermal conductivity  
 $m$ : Mass flow rate  
 $v$ : Volumetric flow rate  
 $\Delta p$ : Differential pressure loss  
 $Q$ : Heat transfer  
 $T$ : Temperature  
 $T_0$ : Reference temperature, 273 K  
 $u$ : Velocity of flow at inlet

