ICMIEE-PI-140395 Fundamental Solution of 3D Transversely Isotropic Elastic Material

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ABSTRACT

A transversely isotropic elastic material is one which physical properties are symmetric about an axis normal to a plane of isotropy. A large number of joints in advanced electronic devices are carried out with the application of these materials. In this paper, an expression of 3D transversely isotopic elastic material is derived. An algorithm of fundamental equation is developed to check the continuity at interface of bonded joint. The Green's function is used for the fundamental solution of three-dimensional transversely isotropic elastic material. Boundary conditions are applied at the interface of the bonded joint and continuity is checked at interface. The effect of displacements(u, v, w), normal stress($\sigma_x, \sigma_y, \sigma_z$), and shear stress ($\tau_{xy}, \tau_{yz}, \tau_{zx}$) with varying distance is analyzed. Finally, united solutions are provided which are suitable for all stable transversely isotropic and isotropic materials.

Keywords: Fundamental Solution, Transversely Isotropic, Bonded Joint, Green's Function

1. Introduction

Elastic material is one which can regain its shape and size after the force applied from that material. Fundamental solutions or Green's functions play an important role in both applied and theoretical studies on the mechanics of solids. Fundamental solutions can be used to construct many analytical solutions of practical problems when boundary conditions are imposed [1]. As a useful calculation approach in engineering science, the boundary element method has been greatly developed in recent years. One of the most important points in the theory of the boundary element method is the point force solution or Green's function [2]. Hu [3] obtained the fundamental solutions for an infinite transversely isotropic solid when $s_1 \neq s_2$, and Pan and Chou [4] also obtained the united solutions for an infinite transversely isotropic solid when $s_1 \neq s_2$ and $s_1 = s_2$, but with different forms of constants. M. S. Islam, H. Koguchi [5] analyzed the intensity of singular stress fields in three-dimensional transversely isotropic piezoelectric bonded joints. Elliott [6] obtained the solution for an infinite solid applied at a point force parallel to the plane of isotropy. Kroner [7] obtained the solution for an infinite solid applied at a point force parallel to the plane of isotropy. This paper present the simplest expression of displacement, shear stress and normal stress of 3D transversely isotropic elastic material and a united solution for transversely isotropic and isotropic material. Therefore, the purpose of this study is to check the continuity of 3D transversely isotropic elastic bi-material joints.

2.1 Basic equations

In the absence of body forces, the governing equation of three dimensional elastic materials is;

$$\sigma_{ij,j} = 0 \tag{1}$$

Where σ_{ij} is stress tensor. These equations are the elastic equilibrium equation. Constitutive relation for elastic material is expressed by Hooke's law;

$$\sigma_{ij} = c_{ijkl} \varepsilon_{kl} \tag{2}$$

Where, ε_{kl} is strain tensor, & c_{ijkl} is elastic constant for material. The elastic strain displacement in Cartesian coordinate $u_i(i = 1,2)$ are related to the strain by the following relation

$$\varepsilon_{ij} = \frac{1}{2} (u_{i,j} - u_{j,i}) \tag{3}$$

Where, $u_{i,j}$ is displacement vector. The characteristics equation of the transversely isotropic elastic material is obtained by solving the given equation.

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$$C_{33}C_{44}s^4 - \left[C_{11}C_{33} + C_{44}^2 - (C_{13} + C_{44})^2\right]s^2 + C_{11}C_{44} = 0$$

Square root of the eigen-equation is denoted by S_1 , S_2 and S_3 is given as below,

$$\begin{split} S_{1} &= \sqrt{\frac{(\bar{C}_{13} - C_{13})(\bar{C}_{13} + C_{13} + 2C_{44})}{4C_{33}C_{44}}} \\ &+ \sqrt{\frac{(\bar{C}_{13} + C_{13})(\bar{C}_{13} - C_{13} - 2C_{44})}{4C_{33}C_{44}}} \\ S_{2} &= \sqrt{\frac{(\bar{C}_{13} - C_{13})(\bar{C}_{13} + C_{13} + 2C_{44})}{4C_{33}C_{44}}} \\ &- \sqrt{\frac{(\bar{C}_{13} + C_{13})(\bar{C}_{13} - C_{13} - 2C_{44})}{4C_{33}C_{44}}} \\ S_{3} &= \sqrt{\frac{C_{66}}{C_{44}}} \qquad [\text{Where, } \bar{C}_{13} = \sqrt{C_{11}C_{33}}] \end{split}$$

2.1 General Solutions:

The general solution for the governing equations of 3D transversely isotropic elasticity has been given by Hu.

When force acting on bonded material, the displacement equations u, v, w is given as follows at $s_1 \neq s_2$

$$u = \sum_{i=1}^{2} \frac{\partial \varphi_i}{\partial x} + \frac{\partial \varphi_3}{\partial y}; \quad v = \sum_{i=1}^{2} \frac{\partial \varphi_i}{\partial y} - \frac{\partial \varphi_3}{\partial x}; \quad w = \sum_{i=1}^{2} \alpha_i \frac{\partial \varphi_i}{\partial z_i}$$

where, $\alpha_1 = \frac{c_{11} - c_{44} s_1^2}{(c_{13} + c_{44}) s_1}$ and $\alpha_2 = \frac{c_{11} - c_{44} s_2^2}{(c_{13} + c_{44}) s_2}$

If φ is the displacement functions for transversely isotropic material then the normal and shear stress equation at $s_1 \neq s_2$ is given as follows;

$$\sigma_{x} = c_{66} \left[\sum_{i=1}^{2} k_{1i} \frac{\partial^{2} \varphi_{i}}{\partial z_{i}^{2}} + 2 \left(\sum_{i=1}^{2} \frac{\partial^{2} \varphi_{i}}{\partial x^{2}} + \frac{\partial^{2} \varphi_{3}}{\partial x \partial y} \right) \right]$$

$$\sigma_{y} = c_{66} \left[\sum_{i=1}^{2} k_{1i} \frac{\partial^{2} \varphi_{i}}{\partial z_{i}^{2}} + 2 \left(\sum_{i=1}^{2} \frac{\partial^{2} \varphi_{i}}{\partial y^{2}} - \frac{\partial^{2} \varphi_{3}}{\partial x \partial y} \right) \right]$$

$$\tau_{xy} = 2c_{66} \left(\sum_{i=1}^{2} \frac{\partial^{2} \varphi_{i}}{\partial x \partial y} - \frac{\partial^{2} \varphi_{3}}{\partial x^{2}} + \frac{\partial^{2} \varphi_{3}}{\partial y^{2}} \right)$$

$$\tau_{zx} = \sum_{i=1}^{2} \omega_{i} \frac{\partial^{2} \varphi_{i}}{\partial x \partial z_{i}} + \omega_{3} \frac{\partial^{2} \varphi_{3}}{\partial y \partial z_{3}}$$

$$\tau_{yz} = \sum_{i=1}^{2} \omega_{i} \frac{\partial^{2} \varphi_{i}}{\partial y \partial z_{i}} - \omega_{3} \frac{\partial^{2} \varphi_{3}}{\partial x \partial z_{3}}$$

$$\sigma_{z} = c_{66} \sum_{i=1}^{2} k_{2i} \frac{\partial^{2} \varphi_{i}}{\partial z_{i}^{2}} \qquad (4)$$

Where, $\xi_i = k_{1i}c_{66}$; $\vartheta_i = k_{2i}c_{66}$; $k_{3i} = (\alpha_i + s_i)/s_3^2$ $k_{2i} = (c_{33}\alpha_i s_i - c_{13})/c_{66}$; $k_{1i} = (c_{13}\alpha_i s_i - c_{12})/c_{66}$ $\omega_i = k_{3i}c_{66}$; $\omega_3 = c_{44}s_3$; [Where, i = 1,2]

3. Fundamental Solution:

At the interface of the bonded joint consisting of two transversely isotropic material is such that the property of the upper material is equal to that of the property of the lower material. Fundamental solution of transversely isotropic elastic material at the interface of the bonded joint when point force applied at a distance h above the interface in x-direction are as follows:

$$\begin{split} \varphi_{i} &= \frac{D_{i}x}{\overline{R}_{ii} + s_{i}|z - h|} + \sum_{j=1}^{2} \frac{D_{ij}x}{R_{ij} + z_{ij}} \\ \varphi_{3} &= \frac{D_{3}y}{\overline{R}_{33} + s_{3}|z - h|} + \frac{D_{33}y}{R_{33} + z_{33}} \\ \overline{z}_{ii} &= z_{ii} = s_{i}(z - h); \quad z_{ij} = z_{i} + h_{j} = s_{i}z + s_{j}h \\ \overline{z}_{33} &= z_{33} = s_{3}(z - h) \\ \overline{R}_{ii} &= \sqrt{(x^{2} + y^{2}) + s_{i}^{2}(z - h)^{2}} \\ R_{ij} &= \sqrt{(x^{2} + y^{2}) + (z_{i} + h_{j})^{2}} \end{split}$$

$$\begin{split} \bar{R}_{ii}^{*} &= \bar{R}_{ii} + s_i | z - h |; \ R_{ij}^{*} = (R_{ij} + z_{ij}) \\ R_{33} &= \sqrt{r^2 + z_{33}^2}; \ \bar{R}_{33} = \sqrt{r^2 + \bar{z}_{33}^2} \\ \bar{R}_{33} &= R_{33} = \sqrt{(x^2 + y^2) + s_3^2 (z - h)^2} \\ D_1 &= \frac{s_1 (c_{11} - c_{44} s_2^2) T}{4\pi c_{11} c_{44} (s_1^2 - s_2^2)}; \ D_2 &= -\frac{s_2 (c_{11} - c_{44} s_1^2) T}{4\pi c_{11} c_{44} (s_1^2 - s_2^2)} \\ D_3 &= \frac{T}{4\pi c_{44} s_3}; \ A_1 = -A_2 = \frac{(c_{13} + c_{44}) T}{4\pi c_{33} c_{44} (s_2^2 - s_1^2)} \end{split}$$

Where, $\varphi_i \& \varphi_3$ are displacement functions and A_i, D_i , $R_{ij}, \overline{R}_{ii}, R_{33}, \overline{z}_{ii}, z_{ij} \& z_{33}$ are known constants.

$$u = \sum_{i=1}^{2} \left[D_{i} \left(\frac{1}{\bar{R}_{ii}^{*}} - \frac{x^{2}}{\bar{R}_{ii}\bar{R}_{ii}^{*2}} \right) + \sum_{j=1}^{2} D_{ij} \left(\frac{1}{\bar{R}_{ij}^{*}} - \frac{x^{2}}{\bar{R}_{ij}\bar{R}_{ij}^{*2}} \right) \right] + \left[D_{3} \left(\frac{1}{\bar{R}_{33}^{*}} - \frac{y^{2}}{\bar{R}_{33}\bar{R}_{33}^{*2}} \right) + D_{33} \left(\frac{1}{\bar{R}_{33}^{*}} - \frac{y^{2}}{\bar{R}_{33}\bar{R}_{33}^{*2}} \right) \right]$$
(5)
$$v = -xy \left[\sum_{i=1}^{2} \left\{ \frac{D_{i}}{\bar{R}_{ii}\bar{R}_{ii}^{*2}} + \sum_{j=1}^{2} \frac{D_{ij}}{\bar{R}_{ij}\bar{R}_{ij}^{*2}} \right\} - \left\{ \frac{D_{3}}{\bar{R}_{33}\bar{R}_{33}^{*2}} + \frac{D_{33}}{\bar{R}_{33}\bar{R}_{33}^{*2}} \right\} \right]$$
(6)

$$w = -x \sum_{i=1}^{2} \alpha_{i} \left[sign(z-h) \frac{D_{i}}{\bar{R}_{ii}\bar{R}_{ii}^{*}} + \sum_{j=1}^{2} \frac{D_{ij}}{R_{ij}R_{ij}^{*}} \right]$$
(7)

$$\sigma_{\rm x} = \sum_{i=1}^{2} \xi_{i} x \left[\left(\frac{D_{i}}{\overline{R}_{ii}^{3}} + \sum_{j=1}^{2} \frac{D_{ij}}{R_{ij}^{3}} \right) - 2C_{66} \left\{ \sum_{i=1}^{2} \left\{ D_{i} \left(\frac{3}{\overline{R}_{ii} \overline{R}_{ii}^{*2}} - \frac{2x^{2}}{\overline{R}_{ii}^{2} \overline{R}_{ii}^{*3}} - \frac{x^{2}}{\overline{R}_{ii}^{3} \overline{R}_{ii}^{*2}} \right) + \sum_{j=1}^{2} D_{ij} \left(\frac{3}{R_{ij} R_{ij}^{*2}} - \frac{2x^{2}}{R_{ij}^{2} R_{ij}^{*3}} - \frac{x^{2}}{R_{ij}^{2} R_{ij}^{*2}} \right) \right\} - D_{3} \left\{ \left(\frac{1}{\overline{R}_{33} \overline{R}_{33}^{*2}} - \frac{2y^{2}}{\overline{R}_{33}^{2} \overline{R}_{33}^{*3}} - \frac{y^{2}}{\overline{R}_{33}^{3} \overline{R}_{33}^{*2}} \right) + D_{33} \left(\frac{1}{R_{33} R_{33}^{*2}} - \frac{2y^{2}}{R_{33}^{2} R_{33}^{*3}} - \frac{y^{2}}{R_{33}^{3} R_{33}^{*2}} \right) \right\} \right\} \right]$$
(8)
$$\sigma_{\rm y} = \sum_{i=1}^{2} \xi_{i} y \left[\left(\frac{D_{i}}{\overline{R}_{ii}^{3}} + \sum_{j=1}^{2} \frac{D_{ij}}{R_{ij}^{3}} \right) \right]$$

$$-2C_{66}\left\{\sum_{i=1}^{2}\left\{D_{i}\left(\frac{1}{\bar{R}_{ii}\bar{R}_{ii}^{*2}}-\frac{2y^{2}}{\bar{R}_{ii}^{2}\bar{R}_{ii}^{*3}}-\frac{y^{2}}{\bar{R}_{ii}^{3}\bar{R}_{ii}^{*2}}\right)\right\}$$

$$\begin{split} &+ \sum_{j=1}^{2} D_{ij} \left(\frac{1}{R_{ij}R_{ij}^{*2}} - \frac{2y^2}{R_{ij}^2R_{ij}^{*3}} - \frac{y^2}{R_{ij}^3R_{ij}^{**2}} \right) \right\} \\ &- D_3 \left\{ \left(\frac{1}{R_{33}\bar{R}_{33}^{**2}} - \frac{2y^2}{\bar{R}_{33}^2\bar{R}_{33}^{**3}} - \frac{y^2}{\bar{R}_{33}^3\bar{R}_{33}^{**2}} \right) \\ &+ D_{33} \left(\frac{1}{R_{33}R_{33}^{**2}} - \frac{2y^2}{R_{33}^2R_{33}^{**3}} - \frac{y^2}{R_{33}^3R_{33}^{**2}} \right) \right\} \right] \quad (9) \\ \tau_{xy} = 2c_{66}y \left[\sum_{i=1}^{2} \left\{ D_i \left(\frac{2x^2}{\bar{R}_{ii}^2\bar{R}_{ii}^{**3}} + \frac{x^2}{\bar{R}_{ij}^3\bar{R}_{ii}^{**2}} - \frac{1}{\bar{R}_{ii}\bar{R}_{ii}^{**2}} \right) \right\} \\ &+ \sum_{j=1}^{2} D_{ij} \left(\frac{2x^2}{R_{ij}^2R_{ij}^{**3}} + \frac{x^2}{R_{ij}^3\bar{R}_{ij}^{**2}} - \frac{1}{R_{ij}R_{ij}^{**2}} \right) \right\} \\ &+ \left\{ D_3 \left(\frac{4x^2}{\bar{R}_{33}^2\bar{R}_{33}^{**3}} + \frac{2x^2}{\bar{R}_{33}^3\bar{R}_{33}^{**2}} - \frac{1}{\bar{R}_{33}^3} \right) \right\} \\ &+ \left\{ D_3 \left(\frac{4x^2}{R_{33}^2\bar{R}_{33}^{**3}} + \frac{2x^2}{\bar{R}_{33}^3\bar{R}_{33}^{**2}} - \frac{1}{\bar{R}_{33}\bar{R}_{33}^{**2}} - \frac{1}{\bar{R}_{33}^3} \right) \\ &+ D_{33} \left(\frac{4x^2}{R_{33}^2\bar{R}_{33}^{**3}} + \frac{2x^2}{R_{33}^3\bar{R}_{33}^{**2}} + \frac{1}{R_{33}R_{33}^{**2}} - \frac{1}{\bar{R}_{33}^3} \right) \right\} \right] \quad (10) \\ &\tau_{xz} = -\sum_{i=1}^{2} \omega_i \left[sign(z-h)D_i \left\{ \frac{1}{\bar{R}_{ii}\bar{R}_{ii}^{*}} - \frac{x^2}{\bar{R}_{ij}^3\bar{R}_{ij}^{*}} - \frac{x^2}{\bar{R}_{ij}^2\bar{R}_{ij}^{*2}} \right\} \right] \\ &- \omega_3 \left[D_3 sign(z-h) \left\{ \frac{1}{\bar{R}_{33}\bar{R}_{33}^{*}} - \frac{y^2}{\bar{R}_{33}^3\bar{R}_{33}^{*}} - \frac{y^2}{\bar{R}_{33}^2\bar{R}_{33}^{*2}} \right\} \right] \quad (11) \\ &\tau_{yz} = xy \sum_{i=1}^{2} \omega_i \left[D_i sin(z-h) \left\{ \frac{1}{\bar{R}_{ii}^3\bar{R}_{ii}^{*}} + \frac{1}{\bar{R}_{ii}^2\bar{R}_{ii}^{*2}} \right\} \right] \end{aligned}$$

$$+\sum_{j=1}^{2} D_{ij} \left\{ \frac{1}{R_{ij}^{3} R_{ij}^{*}} + \frac{1}{R_{ij}^{2} R_{ij}^{*2}} \right\} \right]$$
$$-\omega_{3} xy \left[D_{3} \sin(z-h) \left\{ \frac{1}{(\bar{R}_{33})^{3} \bar{R}_{33}^{*}} + \frac{1}{(\bar{R}_{33})^{2} \bar{R}_{33}^{*2}} \right\} + D_{33} \left\{ \frac{1}{R_{33}^{3} R_{33}^{*}} + \frac{1}{R_{33}^{2} R_{33}^{*2}} \right\} \right]$$
(12)

$$\sigma_{z} = \sum_{i=1}^{2} \vartheta_{i} x \left[\frac{D_{i}}{\bar{R}_{ii}^{3}} + \sum_{j=1}^{2} \frac{D_{ij}}{\bar{R}_{ij}^{3}} \right]$$
(13)

Fundamental solutions of 3D transversely isotropic elastic material at the interface of the bonded joint when

point force applied at a distance h above the interface in z-direction are as follows:

$$\varphi_{i} = A_{i} sign(z - h) \ln(\bar{R}_{ii} + s_{i}|z - h|) + \sum_{i=1}^{2} A_{ij} \ln(R_{ij} + z_{ij}) \varphi_{3} = 0 u = \sum_{i=1}^{2} \left[sign(z - h) \frac{A_{i}x}{\bar{R}_{ii}\bar{R}_{ii}^{*}} + \sum_{j=1}^{2} \frac{A_{ij}x}{R_{ij}R_{ij}^{*}} \right]$$
(14)

$$v = \sum_{i=1}^{2} \left[sign(z-h) \frac{A_i y}{\bar{R}_{ii} \bar{R}_{ii}^*} + \sum_{j=1}^{2} \frac{A_{ij} y}{R_{ij} R_{ij}^*} \right]$$
(15)

$$w = \sum_{i=1}^{2} \alpha_i \left[\frac{A_i}{\overline{R}_{ii}} + \sum_{j=1}^{2} \frac{A_{ij}}{R_{ij}} \right]$$
(16)

$$\tau_{xy} = 2c_{66} xy \sum_{i=1}^{2} \left[A_i sign(z-h) \left\{ \frac{1}{\bar{R}_{ii}{}^3 \bar{R}_{ii}{}^*} + \frac{1}{\bar{R}_{ii}{}^2 \bar{R}_{ii}{}^{*2}} \right\} - \sum_{j=1}^{2} A_{ij} \left\{ \frac{1}{\bar{R}_{ij}{}^3 \bar{R}_{ij}{}^*} + \frac{1}{\bar{R}_{ij}{}^2 \bar{R}_{ij}{}^{*2}} \right\} \right]$$
(17)

$$\tau_{yz} = -\omega_i y \sum_{i=1}^{2} \left[\frac{A_i}{\bar{R}_{ii}^{\ 3}} - \sum_{j=1}^{2} \frac{A_{ij} y}{R_{ij}^{\ 3}} \right]$$
(18)

$$\tau_{zx} = -\omega_i x \sum_{i=1}^{2} \left[\frac{A_i}{\bar{R}_{ii}}^3 - \sum_{j=1}^{2} \frac{A_{ij} x}{\bar{R}_{ij}}^3 \right]$$
(19)

$$\sigma_{x} = -\xi_{i} \sum_{i=1}^{2} \left[\frac{A_{i} \overline{z}_{ii}}{\overline{R}_{ii}^{3}} + \sum_{j=1}^{2} \frac{A_{ij} z_{ij}}{R_{ij}^{3}} \right] + 2c_{66} \sum_{i=1}^{2} \left[A_{i} sign(z-h) \left\{ \frac{1}{\overline{R}_{ii} \overline{R}_{ii}^{*}} - \frac{x^{2}}{\overline{R}_{ii}^{2} \overline{R}_{ii}^{*2}} - \frac{x^{2}}{\overline{R}_{ii}^{3} \overline{R}_{ii}^{*}} \right\}$$

$$+\sum_{j=1}^{2} A_{ij} \left\{ \frac{1}{R_{ij}R_{ij}^{*}} - \frac{x^{2}}{R_{ij}^{2}R_{ij}^{*2}} - \frac{x^{2}}{R_{ij}^{3}R_{ij}^{*}} \right\}$$
(20)

$$\sigma_{y} = -\xi_{i} \sum_{i=1}^{2} \left[\frac{A_{i} \bar{z}_{ii}}{\bar{R}_{ii}^{3}} + \sum_{j=1}^{2} \frac{A_{ij} z_{ij}}{\bar{R}_{ij}^{3}} \right] + 2c_{66} \sum_{i=1}^{2} \left[A_{i} sign(z-h) \left\{ \frac{1}{\bar{R}_{ii} \bar{R}_{ii}^{*}} - \frac{y^{2}}{\bar{R}_{ii}^{2} \bar{R}_{ii}^{*2}} - \frac{y^{2}}{\bar{R}_{ii}^{3} \bar{R}_{ii}^{*}} \right\} + \sum_{i=1}^{2} A_{ij} \left\{ \frac{1}{\bar{R}_{ij} \bar{R}_{ij}^{*}} - \frac{y^{2}}{\bar{R}_{ii}^{2} \bar{R}_{ii}^{*2}} - \frac{y^{2}}{\bar{R}_{ij}^{3} \bar{R}_{ii}^{*}} \right\}$$
(21)

$$\sigma_{z} = \vartheta_{i} \sum_{i=1}^{2} \left[\frac{A_{i} \bar{z}_{ii}}{\bar{R}_{ii}^{3}} + \sum_{j=1}^{2} \frac{A_{ij} z_{ij}}{R_{ij}^{3}} \right]$$
(22)

Similarly, when force applied in upper material, then displacements, (u', v', w'), normal stress, $(\sigma_x', \sigma_y', \sigma_z')$, and shear stress, $(\tau_{xy'}, \tau_{yz'}, \tau_{xz'})$ of lower material has also been derived. Fundamental solutions of 3D transversely isotropic elastic material at the interface of the bonded joint when point force applied at a distance h below the interface have also been derived.

4. Material Properties and Boundary conditions

Transversely isotropic elastic material joint comprising of zinc as upper material and cobalt as the lower material is analyzed for this paper. Elastic constants of the material are tabulated below.

Table 1 Elastic of	constants of	the material
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Material	<i>c</i> ₁₁	<i>c</i> ₁₂	<i>c</i> ₁₃	<i>c</i> ₃₃	<i>C</i> ₄₄	C ₆₆
Zinc (GPa)	158.35	31.5	47.44	61.6	40	63.4
Cobalt (GPa)	307	165	103	358.1	75.3	71

Assume that the two-phase elastic material is perfectly bonded. Therefore the boundary conditions on the interface (z = 0) are given below;

$$\begin{array}{ll} u = u' & v = v' & w = w' \\ \sigma_{zz} = \sigma_{zz}' & \tau_{xz} = \tau_{xz}' & \tau_{yz} = \tau_{yz}' \end{array}$$

Where prime refers to the variables in the lower material, $z \le 0$ and the other ones refers to those in the upper material, $z \ge 0$.

5. Model of 3D transversely isotropic elastic material



Fig.1 Model of 3D bonded joint

Numerical investigation is performed by Green's function with the application of point force at a distance h (50mm) above and below the interface in x, y & z directions. Dimension of upper and lower material is 100mm×100mm×100mm when the value of z varies from -100 mm to 100mm.

4. Result and discussion

Graphical representations of the value of displacement, shear stress and normal stress are given below for both upper and lower material. In all graphs there have a deflection at point 50mm for upper material and -50mm for lower material of the distance, z where the point force is applied. Displacements, (u, v, w) & (u', v', w'), normal stress, $(\sigma_z \& \sigma_z')$ and shear stress, $(\tau_{yz}, \tau_{xz}) \& (\tau_{yz}', \tau_{xz}')$ shows continuity at interface. Normal stress, $(\sigma_x, \sigma_y) \& (\sigma'_x, \sigma'_y)$ and shear stress, $(\tau'_{xy} \& \tau_{xy})$ shows discontinuity at interface of 3D transversely isotropic elastic bi-material joint.



Fig.2 Distribution of displacement (u, v & w) against distance, *z* with force in *z* direction of upper material



Fig.3 Distribution of displacement (u, v & w) against distance, *z* with force in *z* direction of lower material



Fig.4 Distribution of normal stress $(\sigma_x, \sigma_y \& \sigma_z)$ against distance, *z* with force in *z* direction of upper material



Fig.5 Distribution of normal stress $(\sigma_x, \sigma_y \& \sigma_z)$ against distance, *z* with force in *z* direction of lower material



Fig.6 Distribution of Shear stress $(\tau_{xy}, \tau_{yz} \& \tau_{zx})$ against distance, *z* with force in *z* direction of upper material



Fig.7 Distribution of Shear stress $(\tau_{xy}, \tau_{yz} \& \tau_{zx})$ against distance, *z* with force in *z* direction of lower material



Fig.8 Distribution of displacement (u, v & w) against distance, z with force in x direction of upper material



Fig.9 Distribution of displacement (u, v & w) against distance, *z* with force in *x* direction of lower material



Fig.10 Distribution of normal stress $(\sigma_x, \sigma_y \& \sigma_z)$ against distance, *z* with force in *x* direction of upper material



Fig.11 Distribution of normal stress $(\sigma_x, \sigma_y \& \sigma_z)$ against distance, *z* with force in *x* direction of lower material



Fig.12 Distribution of Shear stress $(\tau_{xy}, \tau_{yz} \& \tau_{zx})$ against distance, *z* with force in *x* direction of upper material



Fig.13 Distribution of Shear stress $(\tau_{xy}, \tau_{yz} \& \tau_{zx})$ against distance, *z* with force in *x* direction of lower material

From the graph, the distribution of stress at every point of the bonded material gives the useful information about the permissible amount of force that the material can withstand before its facture. The higher value of displacement and stress occurs at the free edge of the material joint than the inner portion of the joint. The possibility to debond and delamination at interface of the transversely isotropic elastic material joints was due to the higher stress and displacement concentration at the free edge.

5. Conclusion

In this paper, the expression of fundamental equation of 3D transversely isotropic elastic material is derived and algorithm for the fundamental solution is developed by the FORTRAN code. Finally the continuity of the fundamental equation at interface of 3D transversely isotropic elastic material is checked. This analysis clearly shows that the elastic constants and point force have great influence on the material joint.

NOMENCLATURE

: Elastic displacements
: Normal stresses (N/mm ²)
: Shear stresses (N/mm ²)
: Displacement functions
: Known constants
: Undetermined constant
: Dimensionless material parameters
: Point force
: Elastic stiffness tensor
: Characteristic roots

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