Fluid dynamics around two tandem cylinders of different diameters

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ABSTRACT

The paper associated with two tandem cylinders presents the upstream cylinder size (diameter d) effect on global parameters of the downstream cylinder including time-mean drag coefficient (CD), fluctuating drag and lift coefficients (CD and CL), shedding frequencies and flow structures at the spacing ratio L/d = 1.0 ~ 8.0, where L is the distance between the center of the upstream cylinder and the forward stagnation point of the downstream cylinder. d is varied as 8, 16, 24, 32 and 40 mm, while the downstream cylinder diameter D is fixed at 40 mm, corresponding to diameter ratio d/D ranging from 0.2 to 1.0. The Reynolds number is kept constant at 4.27×10^4 based on D. CD, CD and CL are measured using a sectional load cell, while the shedding frequency is estimated from hotwire-measured fluctuating velocity in the wake. Flow structures are obtained using smoke visualization technique. The critical L/d dividing the reattachment and coshedding flows is larger at smaller d/D. CD, CD and CL at the critical spacing leap for d/D = 1.0 - 0.4, but decline for d/D = 0.2. In the coshedding regime, the downstream-cylinder shedding frequency locks-in with the shedding frequency of the upstream cylinder for d/D = 1.0, 0.8 and 0.6, in addition to a subharmonic lock-in for d/D = 0.6 only. The lock-in however does not occur at d/D = 0.4 and 0.2, the shedding frequencies of the upstream and downstream cylinders being much higher and smaller, respectively, compared to the shedding frequency of a single cylinder. CD in general increases with d/D. CD and CL, on the other hand, generally wane and grow as d/D decreases from 1.0 to 0.6 and 0.4 to 0.2, respectively. While the former phenomenon is dominantly influenced by impaired vortices/shear-layer with the decreases in d/D, the later by increased flow velocity between the gap.

Keywords: Tandem arrangement; Flow structures; Fluid forces; Reattachment position

1. Introduction

Flow around two tandem cylinders of identical diameters is in general classified into three major regimes [1]: (i) the extended-body regime (0.5 < L/d < 1.0; d is the upstream cylinder diameter, and L is the distance between the center of the upstream cylinder center and the leading stagnation point of the downstream cylinder); (ii) the reattachment regime (1.0 < L/d < 3.5); (iii) the coshedding regime (L/d > 3.5). There is a transition L/d range between the reattachment and coshedding regimes, where both reattachment and coshedding flows appear intermittently, switching from one to the other. The transition L/d is called critical or bistable flow spacing. Furthermore, the reattachment regime is further divided into two [2]: alternating reattachment regime (1.0 < L/d < 2.5) and steady reattachment regime (2.5 < L/d < 3.5). At the coshedding regime, it is well established that the two cylinders shed vortices separately at the same frequency [2-5]. For a given downstream cylinder, different cross sections of the upstream cylinder with the same characteristic width result in a different frequency of vortex shedding from the upstream cylinder, and the frequency of vortex shedding from the downstream cylinder modifies accordingly to lock-in, adjusting itself to that of the upstream one [6]. On the other hand, for a given upstream cylinder size, a change in the cross section of the downstream cylinder does not influence the shedding frequency of the upstream cylinder [6].

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Fig. 1 (a) Experimental setup and (b) definition of symbols.

It was concluded that the hydrodynamic stability of the flow around two tandem cylinders is predominantly
controlled by the upstream cylinder, not by the downstream cylinder. Measuring time-mean drag coefficient $C_D$, fluctuating drag coefficient $C_D'$, fluctuating lift coefficient $C_L$ and Strouhal number $St$ on two identical cylinders at Reynolds number $Re = 6.5 \times 10^4$ for $L/d < 8.5$, $C_D$, $C_D'$, $C_L$ and $St$ on the downstream cylinder are highly sensitive to $L/d$ particularly in the reattachment regime [2]. Investigations associated with different diameter cylinders are very scarce.

The major objectives of this work are to examine (i) the effect of the upstream cylinder diameter on $C_D$, $C_D'$, $C_L$, $St$ and wake of the downstream cylinder at $L/d = 1 - 8$, covering all possible flow regimes, and (ii) how the critical $L/d$ corresponding to transition between reattachment and coshedding flows is dependent on $d/D$.

2. Experimental setup

Experiments were performed in a closed-circuit wind tunnel with a 5.5 m long test section of 0.8 m × 1.0 m. The downstream cylinder diameter $D$ (= 40 mm) was kept fixed, and the upstream cylinder diameter $d$ was changed as $d = 8, 16, 24, 32$ and 40 mm, resulting in diameter ratio $d/D = 0.2, 0.4, 0.6, 0.8,$ and 1.0, respectively. The spacing ratio $L/d$ between the cylinders was systematically varied from 1.0 to 8.0. The freestream velocity $U_w$ was 16 m/s, corresponding to $Re = 4.27 \times 10^4$ based on $D$. This $Re$ lies in the higher subcritical $Re$ range where fluid forces acting on a single cylinder are comparatively insensitive to $Re$. The turbulence intensity of the uniform flow was 0.45%.

Fluid forces acting on the downstream cylinder are measured using a sectional load cell (65 mm long) installed at the midsection of the cylinder (Fig. 1). Therefore, $C_D$, $C_D'$, and $C_L$ are estimated from the load cell output. Fig. 1 shows schematically the arrangement of the cylinders and definitions of coordinates ($x'$, $y'$) and ($x$, $y$) with the origins defined at the upstream and downstream cylinder centers, respectively. Two single tungsten hotwires, placed in the gap between the cylinders and behind the downstream cylinders, respectively, were used to measure the longitudinal velocity fluctuation $u$ from which vortex shedding frequency was extracted using FFT. The hotwire probe holders were positioned perpendicular to the wake-center plane to minimize the disturbance to the flow.

Smoke-flow visualization technique was used to investigate the flow structure in the gap between two cylinders and behind the downstream cylinder. Pressure measurement on the periphery of the downstream cylinder also carried out, with 32 pressure taps connected to a pressure scanner.

![Reattachment flow](image1.png)  ![Coshedding flow](image2.png)

*Fig. 2 Visualized flow structures at (a) $d/D = 1.0$, (b) $d/D = 0.8$, (c) $d/D = 0.6$, (d) $d/D = 0.4$, and (e) $d/D = 0.2$. Left- and right-column pictures represent reattachment and coshedding flows, respectively.*

3. Results and discussion

3.1 Flow structures and shedding frequency

Figure 2 shows flow structures at different $d/D$ around the two cylinders near critical $L/d$ where the left column displays the photographs corresponding to reattachment flow and the right column to the coshedding flow. Clearly the upstream cylinder shear layers at $d/D = 1.0$ reattach on the downstream cylinder at $L/d = 2.5$ (Fig. 2a1) but roll up in the gap between the cylinders at $L/d = 3.0$ (Fig. 2a2). That is, the critical spacing lies between $L/d = 2.5$ and 3.0. Similarly that at $d/D = 0.8$ and 0.6 nestles between $L/d = 3.0$ and 3.5 (Fig. 2b-c). $d/D = 0.4$ and 0.2, nevertheless, correspond to the critical spacing at $4.0 < L/d < 4.5$ and $6.0 < L/d < 7.0$, respectively. What is interesting here is that with a decrease in $d/D$ from 1.0 to 0.2 the upstream cylinder wake width shrinks gradually; the shrinking is larger in the coshedding regime than the reattachment regime.

The transmutation of flow with $d/D$ and $L/d$ could be further discussed with the aid of power spectral density functions (Fig. 3) of fluctuating velocities obtained in the gap between the cylinders (HT1) and behind the downstream cylinder (HT2). $St (= f_D/U_w, f_D$ is the vortex shedding frequency) jumps from 0.141 to 0.172 between $L/d = 2.5$ and 3.0 for $d/D = 1.0$ and from 0.158 to 0.212 between $L/d = 3.0$ and 3.5 for $d/D = 0.8$. Apparently the jump is due to a drastic modification of reattachment flow to coshedding flow when $L/d$ is increased. While the higher $St$ at the former $d/D$ is smaller than the $St (= 0.197)$ of an isolated cylinder, that at the latter is higher. In the coshedding flow (i.e., $L/d \geq 3.0$) for $d/D = 1.0$, albeit shedding vortices individually,
the two cylinders have an identical \( St \), slightly smaller than the isolated cylinder. This is an established phenomenon for the same diameter tandem cylinders that the convective vortices from the upstream cylinder trigger the shedding from the downstream cylinder, making both \( St \) identical [6]. On the other hand, though having different diameters at \( d/D = 0.8 \), the two cylinders have again identical \( St \), slightly larger than the isolated cylinder. One may arise a question, why is this \( St \) larger? Indeed, now the upstream cylinder diameter is small, \( d = 0.8D \). The shedding frequency from the upstream cylinder is thus larger, given the same approaching flow velocity. \( St \) based on \( d \) can be calculated as \( 0.212 \times 0.8 = 0.17 \) that is almost the same as that at \( d/D = 1.0 \). The downstream cylinder displays two \( St \) (say, 0.134 and 0.268 at \( L/d = 3.5 \)) at \( d/D = 0.6 \) in the coshedding regime, where the upstream cylinder \( St = 0.268 \), suggesting both fundamental and subharmonic locks-in. Since the higher \( St \) of the downstream cylinder fades away at \( L/d = 4.0 \), it is reasonable to assume that the fundamental lock-in perhaps weaker than the subharmonic. At \( d/D = 0.4 \) and 0.2, thought \( St(s) \) of two cylinders are identical in the reattachment regime, they are different in the coshedding regime. The fundamental \( St \) (upstream cylinder, downstream cylinder) = (0.268, 0.134) at \( L/d = 3.5 \) for \( d/D = 0.6 \), (0.397, 0.186) at \( L/d = 4.5 \) for \( d/D = 0.4 \), and (0.795, 0.215) at \( L/d = 7.0 \) for \( d/D = 0.2 \). Both \( St \) augment with \( d/D \). While the augmentation of the upstream cylinder \( St \) with a decrease in \( d/D \) is due to a decrease in diameter, that of the downstream cylinder is caused by a larger flow velocity in the gap between the cylinders. When the upstream cylinder \( St = 0.268 \) at \( (d/D, L/d ) = (0.6, 3.5) \), 0.397 at \( (d/D, L/d ) = (0.4, 4.5) \), and 0.795 at \( (d/D, L/d ) = (0.2, 7.0) \) are normalized by \( d \) instead of \( D \), they becomes 0.1608, 0.159, and 0.159, respectively, comparable to those at \( d/D = 0.8 \) and 1.0.

Fig. 3 The power spectral density functions of hotwire (HT1 and HT2) signals at different \( d/D \) and \( L/d \): (a) \( d/D = 1.0 \), (b) \( d/D = 0.8 \), (c) \( d/D = 0.6 \), (d) \( d/D = 0.4 \), and (e) \( d/D = 0.2 \), at different \( L/d \). Dotted line, HT1; solid line, HT2. The \( St \) is based on \( D \).
Fig. 4 Dependence of Strouhal number $St$ on $L/d$ and $d/D$.

Fig. 5 Distribution of time-averaged pressure coefficient $C_p$ along the surface of the cylinder: (a) $d/D = 1.0$, (b) $d/D = 0.8$, (c) $d/D = 0.6$, (d) $d/D = 0.4$, (e) $d/D = 0.2$.

How the $St$ of the downstream cylinder is influenced by $L/d$ and $d/D$ is shown in Fig. 4. $St$ is sensitive to $L/d$ essentially in the reattachment flow regime, especially for $d/D = 1.0, 0.8, \text{and } 0.6$. $St$ in the reattachment regime increases with $d/D$ decreasing from 1.0 to 0.6. The change in $St$ between $d/D = 0.4$ and 0.2 in the reattachment regime is relatively small. In the coshedding regime, $St$ is greater at $d/D = 0.8$ than $d/D = 1.0$. Its explanation has been given above. But $St$ at $d/D = 0.6$ is much smaller because of the lock-in of the downstream cylinder vortex shedding at the subharmonic frequency of the upstream cylinder shedding. Again a smaller $d/D$ in general corresponds to a higher $St$ between $d/D = 0.4$ and 0.2.

3.2 Pressure distributions and forces

Distributions of pressure coefficient $C_p$ around the surface of the cylinder at various $L/d$ and $d/D$ are shown in Fig. 5. $C_p$ displays a peak at $\theta = 20°-70°$ depending on $d/D$ at the reattachment flow regime and at $\theta = 0°$ at the coshedding flow regime. The peak at the reattachment flow regime represents the shear layer reattachment. $C_p$ at the whole surface in the reattachment regime is negative for $d/D \geq 0.6$, even at the reattachment peak, while that for other $d/D$ is positive at and near the reattachment. At a given $L/d$, $C_p$ at the reattachment increases with a decrease in $d/D$. At the same time, the reattachment position moves toward the forward stagnation point. If it is assumed that the shear layer velocity is the same for all $d/D$, the higher $C_p$ may result from the shear layer reattachment at a smaller angle of incidence which accompany a shift in reattachment position toward the forward stagnation point. In the coshedding regime as well (solid symbols), a smaller $d/D$ accompanies a larger $C_p$ at the forward stagnation point, which further insinuates that the smaller the $d/D$, the larger the flow velocity in the gap between the cylinders. It is worth noting that when a reattachment flow modifies to a coshedding flow, $C_p$ on the base of the cylinder decreases significantly for $d/D = 1.0-0.6$, declines a little for $d/D = 0.4$ and increases considerably for the other $d/D$. 
The reattachment position $\theta_R$ obtained from the $C_p$ distribution is shown in Fig. 6. It seems that while $d/D = 1.0 - 0.6$ have almost the same trend in $\theta_R$, other $d/D (= 0.4$ and $0.2)$ have a different trend, with $\theta_R$ being smaller. There should be a dormant relationship between $\theta_R$ and forces on the cylinder [2]. $C_D$, $C_D'$ and $C_L'$ variations with $L/d$ shown in Fig. 7 reflect that they are strong functions of $L/d$ and $d/D$, especially before the critical spacing. For $d/D = 1.0$ and $0.8$, as $L/d$ increases from $1.0$ to $2.0$, $\theta_R$ precedes and $C_D$, $C_D'$ and $C_L'$ all escalate. At the same $L/d$ range, $d/D = 0.6$ and $0.2$ have the opposite behaviors; $\theta_R$ recedes and $C_D$, $C_D'$ and $C_L'$ all drop. A small increase in $\theta_R$ for $d/D = 1.0$ and $0.6$ between $L/d = 2.0$ and $2.5$ is accompanied by a small decrease in $C_D$, $C_D'$ and $C_L'$. The correlation between $\theta_R$ and forces is, however, weak for smaller $d/D (= 0.4 - 0.2)$. It is remarkable that at the critical spacing $C_D$, $C_D'$ and $C_L'$ all jump for $d/D = 1.0 - 0.4$, but drop for $d/D = 0.2$. The corresponding flow physics is expected to be different, but not known at this stage. $C_D$ in general enhances with a decrease in $d/D$ except at $d/D = 1.0$, $L/d = 2$. $C_D'$ and $C_L'$, on the other hand, generally decreases with $d/D$ decreasing from $1.0$ to $0.6$ and increases with $d/D$ decreasing from $0.4$ to $0.2$. While the former behavior is mostly influenced by vortices/shear-layer weakening with the decrease in $d/D$, the latter by a larger flow velocity between the gap.

A flow map showing reattachment, bistable and coshedding flows is given in Fig. 8. The critical spacing associated with possible bistable flow extends with decrease in $d/D$.

### 4. Conclusions

Fluid dynamics around two tandem cylinders is investigated, where the upstream cylinder size is reduced from $d/D = 1.0$ to $0.2$. Flow structure, pressure distribution, forces and $St$ are paid attention. The main results of this study may be summarized as follows.

1. $St$ is identical for both cylinders in the reattachment regime regardless of $d/D$. In the coshedding regime, the downstream cylinder shedding frequency locks-in to the upstream cylinder for $d/D = 1.0, 0.8$ and $0.6$. 

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**Fig. 6** Variation in reattachment position $\theta_R$ with increase in $L/d$.

**Fig. 7** Dependence on $L/d$ of (a) $C_D$, (b) $C_D'$, and (c) $C_L'$.
At \( d/D = 0.6 \), a lock-in of the downstream cylinder shedding with the subharmonic of the upstream cylinder shedding is also noticed. For the other two \( d/D \), the upstream cylinder shedding frequency is much higher than the downstream cylinder; lock-in is not observed.

2. The critical \( L/d \) dividing the reattachment and coshedding flows is larger at smaller \( d/D \), nesting at 2.5 < \( L/d < 3.0 \), 3.0 < \( L/d < 3.5 \), 4.0 < \( L/d < 4.5 \), and 6.0 < \( L/d < 7.0 \) for \( d/D =1.0, 0.8-0.6, 0.4 \) and 0.2, respectively. At these critical spacing, while \( C_D, C_D' \) and \( C_L' \) leap for \( d/D =1.0 - 0.4 \), they fall for \( d/D = 0.2 \).

3. \( C_D, C_D' \) and \( C_L' \) of the downstream cylinder are very sensitive to \( L/d \) in the reattachment regime, being highly influenced by the shear-layer reattachment position on the downstream cylinder. \( C_D \) in general increases with \( d/D \) except at \( d/D = 1.0, L/d =2.0 \). \( C_D' \) and \( C_L' \) generally diminish and climb with \( d/D \) decreasing from 1.0 to 0.6 and 0.4 to 0.2, respectively. While the former behavior is mostly influenced by vortices/shear-layer weakening with the decrease in \( d/D \), the latter by a larger flow velocity between the gap.

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REFERENCES

**NOMENCLATURE**
- \( d \): upstream cylinder diameter
- \( C_D \): time-mean drag coefficient
- \( C_D' \): fluctuating drag coefficient
- \( C_L' \): fluctuating lift coefficient
- \( L/d \): spacing ratio
- \( D \): downstream cylinder diameter
- \( d/D \): diameter ratio
- \( St \): Strouhal number
- \( Re \): Reynolds number
- \( U_\infty \): the coming flow velocity
- \( u \): local streamwise velocity
- \( f_v \): vortex shedding frequency
- \( C_p \): pressure coefficient
- \( \theta \): azimuthal angle of pressure tap position
- \( \theta_R \): shear layer reattachment position

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