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Implementation of Disaggregation Method in Economic Lot Scheduling of a Jute Industry under Constant Demand

Subrata Talapatra [1] , *Ghazi Abu Taher* [2, *]
[1, 2] Department of Industrial Engineering and Management, Khulna University of Engineering & Technology, Khulna-9203, BANGLADESH

ABSTRACT
This paper deals with the disaggregation problem which refers to a management system for every product solely and establishes a pre-management system of economic lot scheduling. The values of Production time P_t have at the end of the aggregate plan, which is the production capacity in period t. This has to be used to be make several products. While only one product can be made at a time, demand occurs for all the products simultaneously. The available times for all the products have divided. So that, at any point in time, produce one product and build inventory while consuming it and meet the demand for the other products from the built-up inventory. Disaggregation schedule specifies the sizing and timing of production orders for specific items. The objective of any production system is to smooth the production process, enabling uniform production of item over the period.

Keywords: Disaggregation, Economic Lot Scheduling Problem (ELSP), Inventory, Multi-Product, Lot-Sizing and Scheduling, Sequence-Dependent Setups.

1. Introduction

The Economic Lot Scheduling Problem (ELSP) is the problem of finding the production sequence, production times and idle times of several products in a single facility (machine) on a repetitive basis. The demands are made without stock outs or backorders and average inventory holding. Setup costs are minimized. If a particular product or an item taken, it produces at the rate P, the demand is at the rate of D, produce for a certain period and consumes while produce. So inventory is built up at the rate of P-D. The built up inventory will consume at a certain number of period, the cycles begins again. In between the period while the inventory consumes and the next cycle begins, this item is not produced. Then produce another item. The next item start producing after the first item. There is a changeover time or setup time between the two items. The items can be produce up to the next cycle and the cycle length is T.

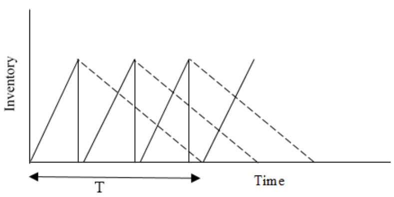

Fig.1 Total cycle time for a particular facility.

In the economic lot scheduling problem, the order of the items is not worried about which one produce first. Once the economic lot scheduling problem is solved whether the order of producing items is made, it will be the same because the cycle time is same for all products. The reason why it is not order dependent because the changeover or setup time depends only the succeeding job and does not depend on the preceding job. It is not defined as the changeover between the two successive jobs. It is defined as the changeover to the job with respect to the earlier product's job. If the changeover time is sequence dependent, which means it depends on both the current job as well as the next job to be done. Then the problem will be more complicated because sequencing them in the order that minimizes the sum of the changeover times.

In the economic lot scheduling problem, it is not assumed that changeover times in sequence dependent. So, when the changeover times are sequence independent, then the economic lot scheduling problem essentially tries to minimize the total cost which is the sum of the ordering cost and carrying cost.

Minimum Total cost (TC) = Ordering cost (OC) + Carrying cost (CC) ----- (1)

While the production is started in a cycle, an inventory is built up during the production and consumes it up to the next cycle. The product which is going to produce first, need no inventory to meet the demand but for the other products, need enough inventory to meet demand before the production period.

In economic lot scheduling problem, the demand (D_j)is the same for every period for each item. It means the demand is same for all periods. The presence of inventory (I_j) ensures that there is no shortage into the cycle. The sufficient inventories meet the demand for each product. The initial inventory increase the cycle length. For the long cycle the ordering cost decreases and the carrying cost increases [1].

* Corresponding author. Tel.: +88-01914883418
E-mail address: ghaziabutaher@yahoo.com

Let's assume, the cycle is long and there is no shortage of time as well as the order cost or setup cost (for manufacturing, order cost = setup cost).

Total cost = Setup cost + Carrying cost ---------------- (2)

2. Literature Review

Numerous research articles have been published in order to include new approaches and extensions to this ELSP problem. Time-varying lot sizes approach which was introduced originally by Maxwell. Matthew proposed another approach which does not require strict regularity of cycle lengths. Zipkin introduced an approach in which some items may be produced repeatedly during a cycle [2]. The different runs of an item can differ in size. Gallego and Roundy extended the time-varying lot sizes approach to the ELSP which allows backorders [3]. Dobson extended his early work by allowing the setup times to be sequence dependent [4]. Gallego and Shaw showed that the ELSP is strongly NP under the time-varying lot sizes approach with or without the Zero Switch Rule (ZSR) restriction, giving theoretical justification to the development of heuristics [5]. Allen modified the ELSP to allow production rates to be decision variables [6]. He developed a graphical method to find the production rates and cycle times for a two product problem. Silver showed that production rate reduction is more profitable for under-utilized facilities [7]. Khouja provided a similar extension for systems with high utilization [8]. Gallego and Moon examined a multiple product factory that employs a cyclic schedule to minimize holding and setup costs [9]. When setup times are reduced, at the expense of setup costs, by externalizing internal setup operations, they showed that dramatic savings are possible for high utilized facilities. Gallego and Moon developed an ELSP with the assumption that setup times can be reduced by a one-time investment [10]. Moon further showed that both setup reduction and quality improvement can be achieved through investment. Khouja used genetic algorithms (GAs) for solving the ELSP which is formulated using the BP approach. Moon developed a hybrid GA based on the time-varying lot sizes approach to solve the ELSP. The stabilization period concept (during which yield rates gradually increase until they reach the target rates) to the ELSP was introduced by Moon [12].

The purposes of this research are to determine the total cycle time T, to find the value of T that optimizes the changeover cost, inventory holding cost for all the periods, to maximize the cycle length, not trying to maximize the total cost, to determine the maximum cycle length at a given inventory [13].

3. Development of Disaggregation Method

In this paper, a disaggregation of economic lot scheduling problem under constant demand and capacity of manufacturing process. For observing the disaggregation problem in manufacturing process, Khalishpur jute mill, Khulna has been visited. They are maintaining an order for their production process. They mainly produce three products such as sacking, Hessian, CBC (Carpet Baking Clothe). The daily demand and the inventory of these products remain constant. The demand and inventory is calculated in man-hours. The demand and inventory data is given in **Table 1**.

Table 1 Day to day demand and inventory data of Khalishpur Jute Mill.

Product	Sacking	Hessian	CBC
Demand	2870	1200	640
Inventory	2030	990	480
r	0.707	0825	0.75

The capacity of the industry is 4710 man-hours. Now the allocation time have to find to make the products.
The value of r represents the demand that can be met with the existing inventory. The production of product j have to start before r_j hours. The products are sorted according to increasing value of r. The order is Sacking-CBC- Hessian. The products are chosen to produce in this order. The process flow depends on the value of r. If process flows from lowest value to highest value of r.

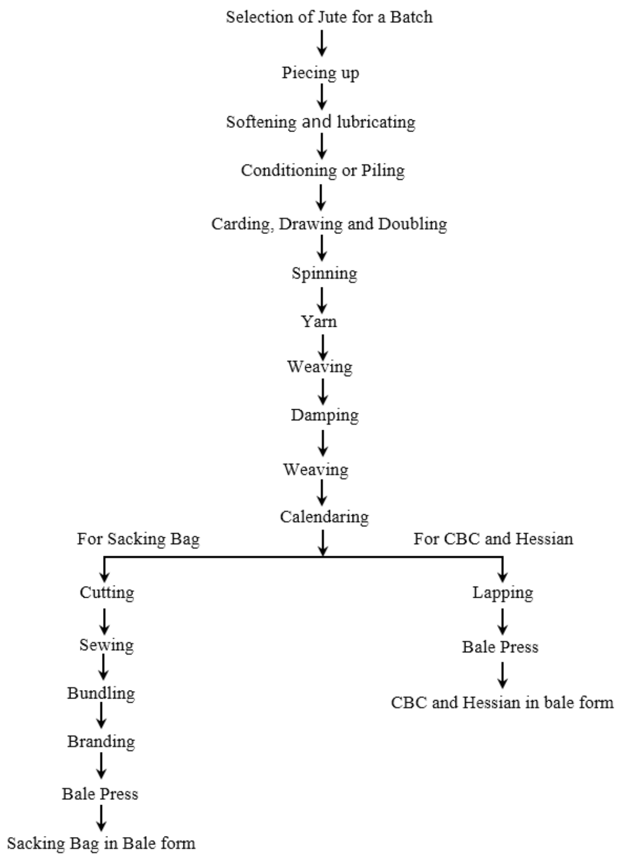

Fig.2 Process flow chart of Khalishpur Jute Mill.

Let t_j be the starting time of production of product j.

Objective function:
Minimize {Setup Cost (SC) + Inventory Cost (IC)}--(3)

The objective function is to minimize the sum of changeover costs and inventory costs. Since the total capacity is equal to the sum of monthly demands and the capacities and demands are assumed to be constant, the amount of inventory will remain same in the system at any point in time. If the inventory holding costs for all products is same, the total inventory cost will be constant.

Set up cost = Set up cost × no of set up = C_o× 3
Inventory cost = Constant

Also, if inventory costs are less compared to changeover costs, the objective shifts to minimizing the number of changeovers, which is to minimize the cycle time T. The objective is to maximize T and the problem reduces to a linear programming problem.

So, Objective function: maximize T

The constraints are,
Subject to,

$$t_j \leq r_j \text{ --------------------------- (4)}$$

constraint for production time for Sacking,

$$(t_{CBC} - t_{Sa})\,4710 \geq \mathrm{T} \times 2870 \text{ ----------- (5)}$$

constraint for production time for CBC,

$$(t_{He} - t_{CBC})\,4710 \geq \mathrm{T} \times 640 \text{----------- (6)}$$

constraint for production time for Hessian,

$$(t_{Sa} + T - t_{He})\,4710 \geq \mathrm{T} \times 1200 \text{ ------------ (7)}$$

constraint for production time limit for Sacking,

$$t_{Sa} \leq 0.707 \text{ ------ (8)}$$

constraint for production time limit for CBC,

$$t_{CBC} \leq 0.75 \text{ --------- (9)}$$

constraint for production time limit for Hessian,

$$t_{He} \leq 0.825 \text{ ----------- (10)}$$

Non-negativity constraints,

$$t_{Sa}, t_{CBC}, t_{He}, T \geq 0$$

Where,
t_{Sa} = Production start time for sacking
t_{He} = Production start time for Hessian
t_{CBC} = Production start time for CBC
T = Total cycle time

The first constraint ensure that the production begins before the inventory of product is exhausted. The second to forth constraints ensure that for every item, the quantity produced in a cycle is enough to meet the demand during the cycle. In constraint (5), $t_{CBC} - t_{Sa}$ represents the time in which Sacking is produced and in constraint (6), $t_{He} - t_{CBC}$ represents the time in which item CBC is produced. But in constraint (7), Sacking is produced in the second cycle at time $t_{Sa} + T$.
Each cycle involves three changeovers, one for each item and we will have 24/T cycles in a day, if the demand and inventory are in terms of man-hours/ day.

4. Algorithm

Algorithm used for solving the problem is given below:

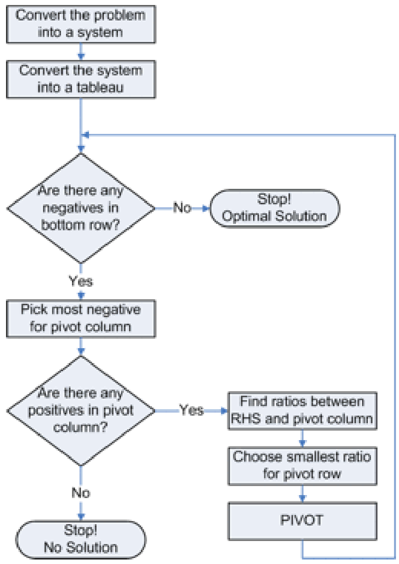

Fig.3 Algorithm of simplex method.

Here TORA software is used for solving the problem. The optimal solution of the problem is given below,
$X_1 = t_{Sa} = 0$
$X_3 = t_{CBC} = 0.6746$
$X_2 = t_{He} = 0.8250$
$X_4 = T = 1.1071$

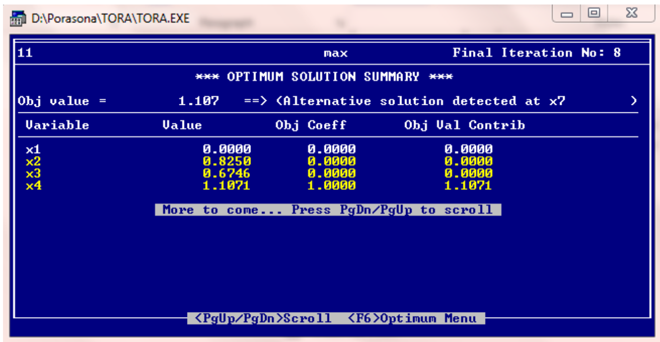

Fig.4 Optimal solution summary of linear programming by using TORA.

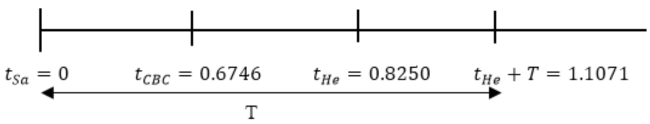

Fig.5 Cycle length.

In this solution, the total inventory in the system at any point is 3500 man-hours. The inventories of various items at the changeover times are given in **Table 2**.

Table 2 Day to day inventory position of Khalishpur Jute Mill.

Time (hours)	Sacking	CBC	Hessian	Total Inventory (man-hours)
t=0	2030	480	990	3500
t=0.6746	3272	48	181	3501
t=0.8250	2841	659	0	3499
t=1.1071	2030	480	990	3500

During the period 0 to 0.6746, Sacking is produced at the rate of 4710 man-hours. Since sacking is also consumed at the rate of 2870 man-hours, its inventory at 0.6746 is {(4710 - 2870)* 0.6746} + 2030 = 3272. CBC is consumed from inventory at the rate of 1200 man-hours. Inventory of Hessian at time = 0.6746 is, therefore, 480 – (640*0.6746) = 48.

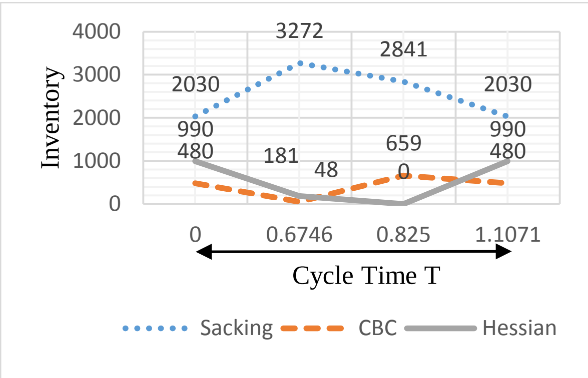

Fig.6 Graphical representation of inventory position of Khalishpur Jute Mill.

5. Increasing T by reallocating the inventory

It is assumed that there is a cycle (called a transient cycle where the inventories are distributed) and a steady state cycle when production started at the stock is zero. The production start time in the transient cycle is denoted by t_j' and the production start time in the steady state cycle is denoted by t_j . The transient and steady state cycles are shown in Fig.7.

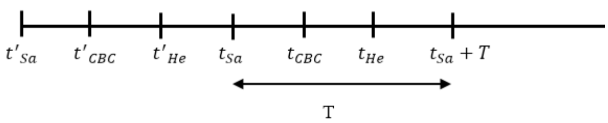

Fig.7 Transient and steady state cycle.

For next cycle t'_{Sa}, t'_{He}, t'_{CBC} are the start times in the first cycle and t_{Sa} ,t_{He}, t_{CBC} are in the steady state cycle.

The formulation becomes;

Objective function: Maximize T

Subject to, $t_j \leq r_j$ ------------------------------- (11)

constraint for production time for Sacking of 1st cycle,

$2030 + 4710\,(t'_{CBC} - t'_{Sa}) \geq 2870\, t_{Sa}$-------- (12)

constraint for production time for CBC of 1st cycle,

$480 + 4710\,(t'_{He} - t'_{CBC}) \geq 640\, t_{CBC}$ ----------- (13)

constraint for production time for Hessian of 1st cycle,

$990 + 4710\,(t'_{Sa} - t'_{He}) \geq 1200\, t_{He}$ -------- (14)

constraint for production time for Sacking of 2nd cycle,

$4710\,(t_{CBC} - t_{Sa}) \geq 2870\, T$ -------- (15)

constraint for production time for CBC of 2nd cycle,

$4710\,(t_{He} - t_{CBC}) \geq 640\, T$ ------------ (16)

constraint for production time for Hessian of 2nd cycle,

$4710\,(t_{Sa} + T - t_{He}) \geq 1200\, T$ -------- (17)

constraint for production time limit for Sacking of 2nd cycle, $t'_{Sa} \leq 0.707$ ---------------- (18)

constraint for production time limit for CBC of 2nd cycle, $t'_{CBC} \leq 0.75$ --------------------- (19)

constraint for production time limit for Hessian of 2nd cycle, $t'_{He} \leq 0.825$ ---------------- (20)

Non-negativity constraints,

$$t_j',\ t_j,\ T \geq 0$$

The first constraint ensures that the production in the transient cycle should start before the inventory is exhausted. Constraint (12) to (14) ensures that the production in the transient cycle plus inventory meets the demand up to the steady state production. Constraints (15) to (17) ensure that the steady state production is equal to the cycle demand.

The optimal solution to this linear programming is given below,

$X_7 = T = 2.7253$

$X_1 = t'_{Sa} = 0$

$X_2 = t'_{CBC} = 0$ [the value is very close to zero]

$X_3 = t'_{He} = 0.2199$

$X_4 = t_{Sa} = 0.7073$

$X_5 = t_{CBC} = 2.3680$

$X_6 = t_{He} = 2.7383$

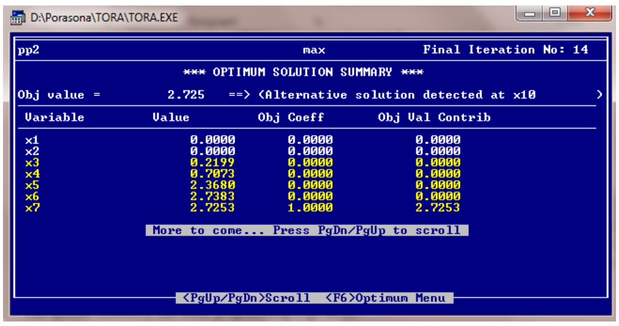

Fig.8 Optimal solution summary of linear programming by using TORA.

The next cycle starts with sacking being produced at time = 0.7073 + 2.7253 = 3.4326

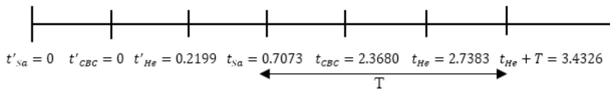

Fig.9 Total Cycle length.

The inventory position at the various time periods are shown in **Table 3**.

Table 3 Day to day inventory position of Khalishpur Jute Mill.

Time	Sacking	CBC	Hessian	Total Inventory
t = 0	2030	480	990	3500
t = 0.2199	1983	1517	0	3500
t = 0.7073	0	1035	2465	3500
t = 2.3680	3056	0	445	3501
t = 2.7383	1992	1507	0	3499
t = 3.3426	0	1035	2465	3500

The above model shows how the cycle time T can be extended by reallocating the inventory such that the products are made when the inventory becomes zero.
The graphical representation of day to day inventory position of Khalishpur Jute Mill is given below:

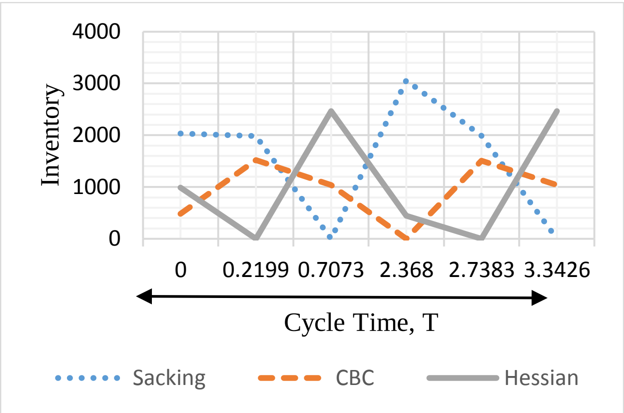

Fig.10 Graphical representation of day to day inventory position of Khalishpur Jute Mill.

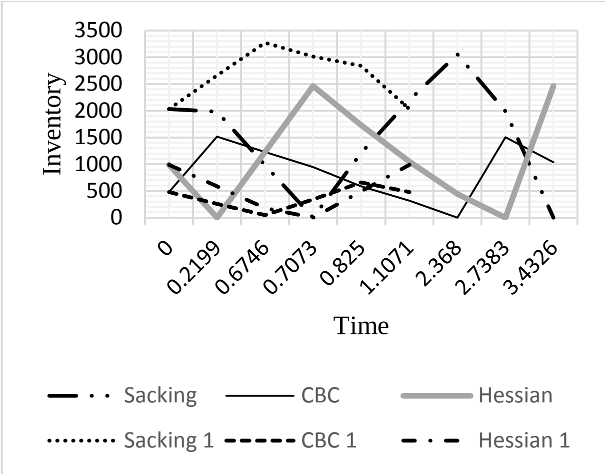

Fig.11 Graphical representation of inventory position of Khalishpur Jute Mill for different cycle time.

Result and Discussion

At the moment of steady cycle begins, sacking is produced when the inventory is zero. At the time 2.3680, when CBC start producing, the inventory is zero and so on for the hessian. But for the first model, when sacking start produced the inventory is not zero. The inventory is zero only for hessian. So, at steady state cycle the inventory is adjusted in such a manner that the values are zero which is able to increase the cycle time as long as we can. At the beginning of next cycle the inventory will be the same. From the table, the total inventory is 3500 for any period because the total demand and the total production is equal which is 4710. In the second cycle, total inventory is only redistributed among the products so that the cycle time can increase as large as possible. There are two aspects are in this problem. One is the cycle time T that depends on the total amount of inventory. If total demand is equal to the total production the cycle time depends on the total inventory. Secondly and more importantly it also depends on the way in which the existing inventory is distributed. So, the existing inventory is distributed in such a manner that the cycle time T exceeds.

Conclusion

This model help us to understand the concept of disaggregation to allocate the available time to the various products. In the second model the objective function is to maximize the cycle length that makes zero inventories at the starting point of each product whereas in the first model there is a buffer inventory at the starting point of each product. So, if the demand is purely deterministic and it does not going to change at all then one can take the risk of redistributing the inventory (like second model) in such a manner that one can achieve maximum cycle time.
The limitation of the second model is that it only works when the demand is constant and continuous. If the demand is not continuous and showing variations then there is a shortage in the cycle. To remove the shortage, a safety stock is put over the model otherwise use the

first model so that the cycle stretch solely at the last point and keep a balance of inventory of all other points. The cycle length depends both the amount of inventory and the way of distributing the inventory. If someone wants to make the cycle length smaller, he has to make the amount of inventory small. If the organizations follow the rules of zero inventories or lesser inventory, the optimization of the problem automatically leads to the smaller values of cycle time. Another dimension to it is also to seen that the tradeoff between inventory holding cost and the changeover cost. If the changeover times are smaller, then automatically more change overs are possible and also able to make more variety of product keeping the cycle time as small as possible.

NOMENCLATURE

n	:	no. of products or items in a cycle = 1, 2, 3… m
I_j	:	Inventory of each of the item on hand (man-hours), where j = 1, 2, 3… m
D_j	:	Demand of each of the item (man-hours), where j = 1, 2, 3… m
P_j	:	Production rate of the item (man-hours), where j = 1, 2, 3… m
T	:	Cycle length (hours)
r	:	ratio of inventory and demand, hours = $\frac{Inventory}{Demand}$

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