ICMIEE-PI-140358

Analysis of Bio-heat Transfer Problem Using Finite Element Approach

A M M Mukaddes, Md. Atiqur Rahman and Abdullah Al Razi
Department of Industrial and Production Engineering, SUST, Sylhet-3114
Bangladesh
Email: mukaddes1975@gmail.com

ABSTRACT

Bio-heat transfer is the study of external or internal heat transfer in the biological body. In different therapeutic treatments especially in cancer treatment, heat is used to cure infected cells. The required temperature that will kill the infected cell should be known before starting the thermal treatment on human tissue. The useful ways to measure the temperature distribution on human tissue are finite difference method as well as analytical method in some cases. There are few reports of finite element solution of bio-heat equation in the literature. Though finite element approach is one of the efficient techniques to find the temperature distribution in physical body, there have been very few reports of using this method for this particular issue. In this paper a finite element model has been developed to analyze the bio-heat transfer equation which is also well known as Pennes equation. Crank-Nicolson method has been used for the time discretization of the unsteady part of the problem. The developed system can be used to predict the temperature in human tissue under certain external heating conditions.

Keywords: Finite element, bio-heat, Pennes equation, unsteady problem

1. Introduction

Heat transfer is an important component of biological activities. For example, skin plays a variety of important roles including sensory, thermoregulations and body defense etc. Amongst these, the most important one is thermoregulation: skin functions thermally as a heat generator, absorber, transmitter, radiator, conductor and vaporizer, thus acting as an important barrier for the human body to various outside conditions. Except some environmental conditions skin/tissue has also to face some experimental conditions during some therapeutic treatments. Cancer treatment, treatment of burn injuries and diagnosis of other thermal diseases requires the information of temperature distribution in living tissue. Therefore studying of the bio-heat transfer has become popular in the bio-mechanical engineering research area.

In some therapeutic treatment, heat is used to kill or remove the infected cell. For example, the primary objective of the hyperthermia is to raise the temperature of the infected cell to a therapeutic value, typical $42\text{-}46^0$ C, and then deactivate it thermally.

Over the years, several mathematical models have been developed to describe the heat transfer within living biological tissues. These models have been widely used in the analysis of hyperthermia in cancer treatment, laser surgery, thermal comfort treatments, and many other applications. Many experimental and theoretical studies have confirmed the important role played by thermal processes during hyperthermia [1]. The most widely used bio-heat model was introduced by Pennes in 1948 [2]. It is based on the classical Fourier law, and has been greatly simplified after introducing the concept of blood perfusion to study the bio-heat transfer and assessment of skin burns. Reports on analytical solution and numerical solutions of the bio-heat transfer problem are found in the literature. They can be distinguished by the boundary conditions used on the skin surface, blood perfusion, heat flux on the skin surface and solution techniques. In some analytical cases sinusoidal heat flux [3] and sometimes cooling of the skin [4] are considered as boundary conditions. Researches related to the bio-thermo-mechanical are reviewed in [5]. The result of boundary element method and finite element method for the numerical solution of the steady-state bio-heat transfer model of the human eye are compared in [6, 7]. The finite element method is used for the thermal-magneto static analysis in biological tissues in [8].

ADVENTURE_Thermal is a parallel FEM open source module to solve the large scale 3D heat

E-mail address: mukaddes1975@gmail.com

conduction problems. The module considers the general heat conduction equation for the solid body. It supports different types of boundary conditions. Both steady and unsteady problem can be solved using this module. The hierarchical domain decomposition method is used to solve the problems in parallel computer. Development of ADVENTURE_Thermal is a continuing process. But this module cannot be used to find temperature distribution in biological bodies. The inclusion of bio-heat transfer functions in this module is necessary.

In this paper a finite element model [9, 11] has been developed for the numerical solution of the 1D unsteady Pennes equations with different spatial heating. The Crank-Nicolson method is used for the time discretization. This method is compared with other time discretization schemes. A C code has been developed which can be used to measure the temperature distribution in the human tissue. Later the function for the 3D bio-heat equation solution will be included in the ADVENTURE_Thermal.

The bio-heat transfer equation is introduced in the section 2 while section 3 describes the finite element discretization method. The time discretization scheme is explained in section 4. Before the conclusion some numerical results are shown in section 5.

2. Bio-heat transfer

For the study of bio-heat transfer in human tissue the most useful one is Pennes equation which is:

$$\rho c \frac{dT}{dt} = k\frac{d^2T}{dx^2} + w_b \rho_b c_b\,(T_a - T) + Q_m + Q_r(x,t) \qquad (1)$$

where ρ, c, k are respectively the density, the specific heat, and the thermal conductivity of the tissue; ρ_b, c_b denote density and specific heat of blood; w_b the blood perfusion; T_a the known arterial temperature, and $T(x,t)$ is unknown tissue temperature; Q_m is the metabolic heat generation, and $Q_r(x,t)$ the heat source due to spatial heating with respect to time t. Specific heat and density of both blood and tissue are changeable with respect to conditions. Natural and force heat convection coefficient between skin and surroundings can also be changed with change of temperature, air flow, postures etc.

For a one dimensional problem of length L Let, $T(x,0) = T_0(x)$ is initial temperature, T_c is the body core temperature and often regarded as a constant, h_0 is the apparent heat convection coefficient between the skin surface and the surrounding air, T_f is the surrounding air temperature. Thus the boundary conditions for this particular 1-D problem can be written as:

$$T = T_0(x) = T_c\,, \qquad at\;\; x = L$$

$$-k\frac{dT_0(x)}{dx} = h_0\left[T_f - T(x)\right],\;\; at\;\; x = 0 \qquad (2)$$

Here, the skin surface is defined at $x = 0$ and the body core at $x = L$.

The analytical solution of the differential equation (1) with boundary conditions of equation (2) was developed by Zong-Shan Dang and Jing Lui [10]. The bio-heat equations (1) and (2) have also been solved using the finite difference method and boundary element method. In this paper the finite element approach is developed to solve the bio-heat equation (1) and (2).

3. Finite element discretization

The first step of the finite element discretization is to develop a weak form that is a weighted-integral statement and is equivalent to both the governing differential equation as well as certain type of boundary conditions. The simplest form of the equation (1) is

$$\rho c\frac{dT}{dt} = k\frac{d^2T}{dx^2} - CT + q \qquad (3)$$

where, $C_b = w_b \rho_b c_b$

and $q = C_b T_a + Q_m + Q_r(x,t)$.

The weak form [9] of the differential equation (3) was derived as

$$\int_{x_a}^{x_b}\left[W\rho c\frac{dT}{dt} + k\frac{dW}{dx}\frac{dT}{dx} + CWT - Wq\right]dx + (WQ)_{x_a} + (WQ)_{x_b} = 0\;. \qquad (4)$$

where W is the weight function and Q is the secondary variable.

In this paper, a linear element is considered whose temperature function is given as:

$$T_h^e(x) = \sum_{j=1}^{2} \varphi_j^e(x) T_j^e \qquad (5)$$

Using the linear approximation of equation (5) finally a linear equation was derived [9]. That is:

$$[C]\{\dot{T}\} + [K]\{T\} = \{q\} + \{Q\} \qquad (6)$$

where C is the capacitance matrix, K is heat conductive matrix and T is unknown temperature and others are known vectors. This set of linear equation was solved using the well-known time discretization method.

Corresponding author: mukaddes1975@gmail.com

4. Time Discretization Scheme

A simple time integration scheme for equation (6) was derived by assuming that C and K are constant. In such case matrix differential equation can be discretized on time [11] as:

$$C\frac{T^{n+1}-T^{n}}{\Delta t}+\alpha K T^{n+1}+(1-\alpha)KT^{n}=q+Q \quad (7)$$

where T^{n+1} and T^{n} are the vectors of unknown nodal values at times $n\Delta t$ and (n+1) Δt respectively. α is a weighting factor which must be chosen in the interval between 0 and 1. In equation (7) the standard approximation for time derivative

$$\dot{T}=\frac{T^{n+1}-T^{n}}{\Delta t}$$

was used. When the value of α is considered 0.5, the process is called the popular Crank-Nicolson method.

The discretized equation (7) can be written as:

$$\left(\frac{1}{\Delta t}C+\alpha K\right)T^{n+1}=\left[\frac{1}{\Delta t}C-(1-\alpha)K\right]T^{n}+Q+q \quad (8)$$

and can also be written in the general form:

$$HT^{n+1}=F^{n} \quad (9)$$

where $H=\left(\frac{1}{\Delta t}C+\alpha K\right)$ and

$$F^{n}=\left[\frac{1}{\Delta t}C-(1-\alpha)K\right]T^{n}+q+Q$$

The equation (8) or (9) was solved using an iterative procedure. The initial temperature is known and then temperature of the next step can be calculated from the solution of equation (8) via the Gauss elimination technique.

5. Analysis and Numerical Results

A FEM code has been developed using the C language to solve the numerical solution of the finite element model described in the previous section. Both steady and unsteady state results are presented here.

Problem statement:

In this paper, a tissue of length 3 cm from the skin surface is considered for the calculation. The convection boundary conditions and temperature boundary conditions are considered on the skin surface and body core respectively. The tissue properties and parameters of the boundary conditions (Table 1) are applied as given in [10]. Both steady and unsteady problems are analyzed. Analytical results and numerical results for different time step are reported.

Steady state results

Steady problem is analyzed first. For FEM analysis 21 meshing nodes and 20 linear elements are considered. The temperature profile for 21 nodes is shown in Figure 1. The steady state results are compared with the analytical solution developed by Zong-Shan Dang and Jing Lui [10]. The comparative results are shown in Table 2 for the mesh of 21 nodes. According to the Table 2, the FEM results match with the analytical results.

Table 1 Thermo mechanical properties of tissue

Symbols	Values
k	0.5 W/m ^{0}C
h_0	10 W/m^2 °C
h_f	100 W/m^2 °C
T_f	25 °C
T_a	37 °C
T_c	37 °C
Q_m	33800 W/m^3.
C	4000 J/kg.°C
c_b	4200 J/kg.°C
P	1000 kg/m^3
ρ_b	1000 kg/m^3
W_b	0.0005 ml/s/ml

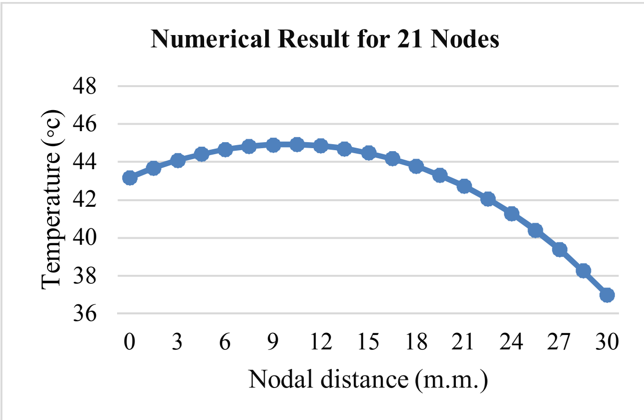

Figure- 1 Steady state results (21 nodes)

Corresponding author: mukaddes1975@gmail.com

Table-2 FEM and analytical solution

Dist. (m.m.)	Temp.(◦C) (Ana.)	Temp. (◦C) (FEM)
0	43.1913	43.1965
1.5	43.6912	43.6964
3	44.1021	44.1074
4.5	44.428	44.4332
6	44.672	44.6771
7.5	44.8363	44.8413
9	44.9224	44.9274
10.5	44.9313	44.9362
12	44.863	44.8677
13.5	44.7168	44.7213
15	44.4909	44.4957
16.5	44.1846	44.1887
18	43.7935	43.7974
19.5	43.3145	43.3181
21	42.743	42.7462
22.5	42.0736	42.0764
24	41.2999	41.3024
25.5	40.4147	40.4166
27	39.4095	39.4108
28.5	38.2749	38.2755
30	37	37

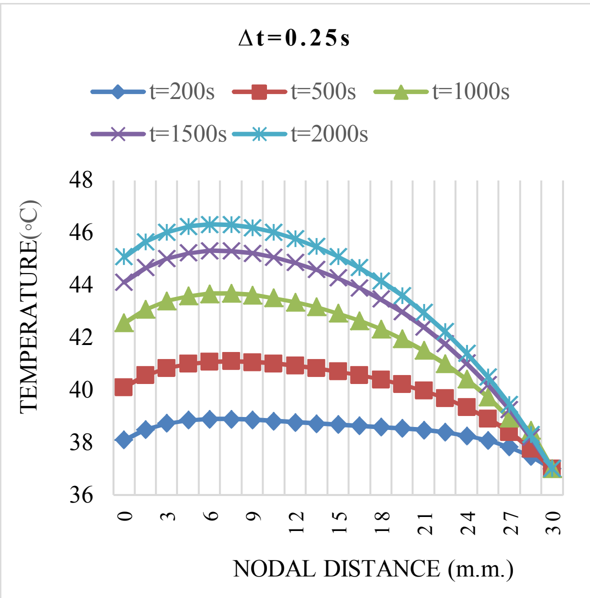

Figure-2 Temperature at different time ($\Delta t = 0.25s$)

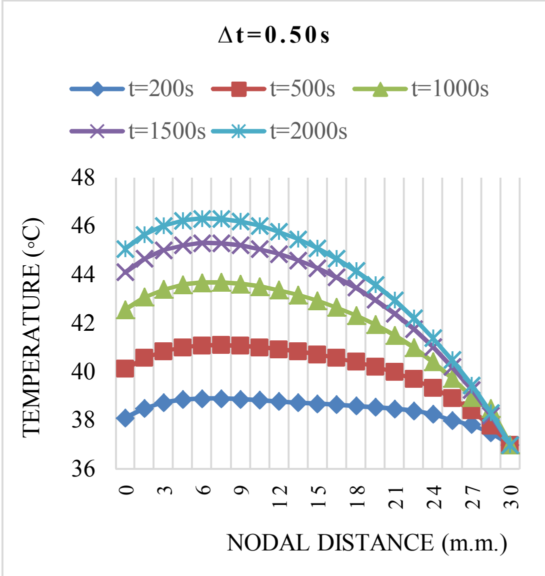

Figure-3 Temperature at different time ($\Delta t = 0.5s$)

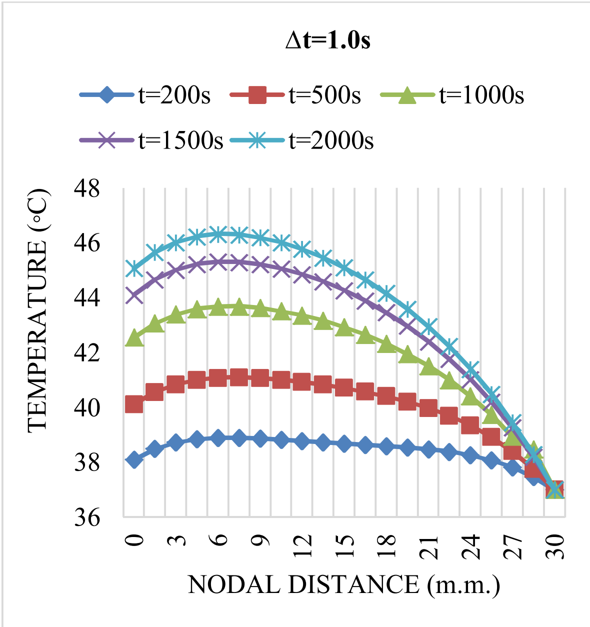

Figure-4 Temperature at different time ($\Delta t = 1.0s$)

Unsteady state results

In the unsteady state analysis different time increments are used to measure the temperature at different time. The initial temperature was 37^0C which is equal to the core temperature. The results are presented in the Figure 2, 3, and 4. These figures show that as time passes the temperature within the tissue increases and finally reached to the steady state condition. The results of these figures consider the constant spatial heating from the skin surface to the body core. There is no significant change of temperature distribution due to the change of time increment. The maximum temperature exists 46^0C at 6 mm below the skin surface.

Corresponding author: mukaddes1975@gmail.com

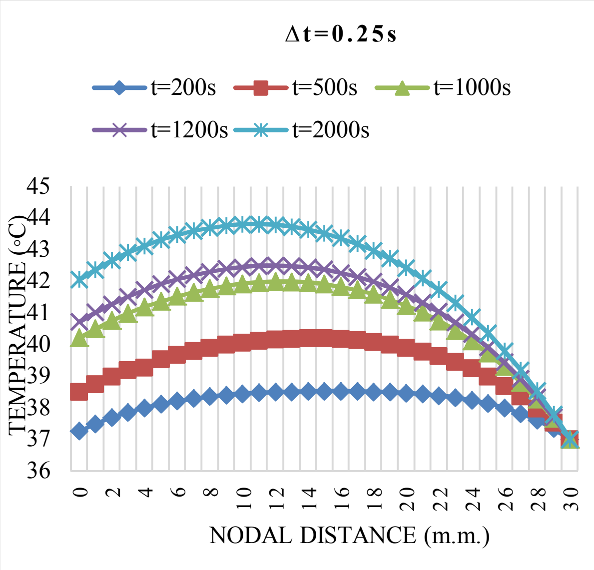

Figure-4 Temperature at different time ($\Delta t = 0.25s$)

A temperature distribution found without the spatial heating is shown in Figure-5 for the time increment ($\Delta t = 0.25s$). The comparison with the results with spatial heating shows that the position of maximum temperature is changed. The maximum temperature exists near the skin surface in case of with spatial heating and near the middle of the model in case of without spatial heating.

In the field of bio-heat transfer, finite element method can be used to design medical equipment for therapeutic applications, where operation can be time dependent or time independent. From this analysis, we have seen how temperature distribution can be changed with respect to time. The effects of time interval and different tissue properties can be analyzed using the developed system. Impacts of spatial heating have also been evaluated from this analysis. Different amount of spatial heating can give different temperature distribution. So a required temperature distribution can be synchronized by different spatial heating and different time limit.

7. Conclusion

A finite element model was developed in this work to analyze 1D steady and unsteady bio-heat transfer in biological tissue. A computer program was also developed using C language and the bio-heat problem was solved using it. Required temperature distribution can be found in human tissue for both steady and unsteady states. The FEM results coincide with analytical results for problem described in the paper. With and without spatial heating conditions are compared. The location of the maximum temperature is changed with the spatial heating. From the analysis, highest value of temperature was found at the distance of 6 m.m. from the skin (where distance from the skin to body core is 30 m.m.), which can also be varied for different conditions. Using different boundary conditions and applying different amount of heat flux one can select the suitable skin condition to maintain the acceptable tissue temperature and destroy the infected cell.

8. Future study

This work concerns with the 1D finite element analysis of bio-heat transfer equation. The coding for 2D bio-heat transfer problem is going on. The spatial heating sometimes acts on sinusoidal function which will be added in the new code. Finally a 3D bio-heat finite element system will be added to the ADVENTURE_Thermal [12] which is a module for steady and unsteady heat conduction problems using the parallel finite element method.

References

1.S. Mandal, T. Sundarajan and P. S. Ghoshdastidar, A coupled analysis of thermal and electromagnetic phenomena during hyperthermia in biological tissues. *ASME winter annual meeting,* HTD-Vol 126, pp. 59-66 (1889).

2. H. H. Pennes, Analysis of Tissue and Arterial Blood Temperatures in the Resting Human Forearm. *Journal of Applied Physiology,* Vol. 1, pp. 93-122 (1948).

3. T. Shih, Y. Ping, W. Lin and H. Kou, Analytical analysis of the Pennes bioheat transfer equation with sinosoidal heat flux condition on the skin surface, *Medical Engineering and Physics*, Vol 29, pp. 946-953, (2007).

4. W. Shen and J. Zhange, Modeling and Numerical Simulation of Bioheat Transfer and Biomechanics in Soft Tissue, *Mathematical and Computer Modeling,* Vol. 41, pp. 1251-1265 (2005).

5. F. Xu, T. J. Lu, K. A. Seffen, Biothermomechanical behavior of skin tissue, *Acta Mechanica Sinica,* Vol. 24, No. 1 pp. 1-23 (2008).

6. E. H. Ooi, W. T. Ang and E. Y. K. Ng, A boundary element model of the human eye undergoing laser thermokeratoplasty, *Computers in Biology and Medicine.* Vol. 38, pp. 727-737, (2008).

7. E. H. Ooi, W. T. Ang and E. Y. K. Ng, Bioheat transfer in the human eye: A boundary element approach, *Engineering Analysis with Boundary Elements,* Vol. 31, No. 6, pp. 494-500(2007).

8. S-C Hwang, and Lemmonier, Coupling numerical solution of bio-heat transfer equation and Maxwells equations in biological tissues during hyper-

Corresponding author: mukaddes1975@gmail.com

thermia, *Transactions on Biomedicine and Health*, Vol 2, pp. 435-443 (1998)

9. Reddy, J. N. An Introduction to the Finite Element Method, 3rd edition, McGraw Hill, Inc, (2006).

10. J. Lui and D. Zhong-Shan, Analytical study on bioheat transfer problems with spatial or transient heating on skin surface or inside biological bodies. *Journal of biomechanical engineering*, Vol-125(6), p. 638-649 (2002).

11. G. Comini, S. D. Giudice and C. Nonino, Finite Element Analysis in Heat Transfer, Taylor and Francis, USA, (1994).

11. http://adventure.sys.t.u-tokyo.ac.jp/

Corresponding author: mukaddes1975@gmail.com