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Modeling Progressive Damage and Failure for Polymer-Matrix Composites

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ABSTRACT

A model to predict the damage evolution and failure in unidirectional fiber-reinforced polymer-matrix composites (PMC) is developed using CCM-Schapery-Crack Band theory. Polymer matrix progressive damage is modeled using Schapery Theory (ST), which is later extended up to failure in order to account for more catastrophic failure mechanisms. The degrading elastic parameters of the fiber-reinforced PMC are obtained as a function of damage to finally determine the amount of damage associated with a PMC under uniaxial, biaxial, multiaxial and combined transverse/axial-shear loading.

Keywords: Polymer Matrix Composites, Damage Mechanics, Finite Element Method, Progressive Damage.

1. Introduction

In the last two decades, the advancement of composite materials has been generating technological and economic improvements. Advanced composite materials have been used in aircrafts, automobiles, industrial machinery, sporting goods and in many other applications. These applications require high-performance constituents, e.g. carbon fibers, glass fibers, polymers, ceramics, metals. Therefore, to assure the efficiency of these composite materials, a progressive damage and failure analysis has to be performed. According Pankow [1] the manufacturing of textile composite materials is becoming more economically feasible. The production of large scale composite structures has been continuously increasing in recent years. Therefore, as stated by Cox and Yang [2] analytical predictions of damage and failure are needed in the early stages of the design, reducing time and money spent on manufacturing the material to test the design.

According Pineda [3] the lay-up and directionality of the load applied to a fiber-reinforced PMC determine the global damage and failure mechanisms generated in the PMC. The extent of damage in PMCs is dependent on various material parameters, of which the fiber volume fraction, matrix and fiber properties along with the type of loading have shown to be critical. The influence of these parameters on the damage in fiber reinforced PMCs have been investigated by several researchers. Most of the non-linearity observed can be attributed to the damage evolution in the polymer matrix. Fiber breaking in PMCs is rather abrupt, whereas other damage mechanisms like microcracking, fiber-matrix debonding, transverse cracking, etc. are progressive in nature. For example, tensile loading in the transverse direction may cause microcracks or voids to grow resulting in transverse cracks[4], Schapery[5]. Interlaminar separation, also known as delamination, may occur when transverse cracks intersect an interface between two adjacent layers within a laminate[6], rendering the laminate weak in shear and transverse loading. Another types of loading is compression where the degradation of the polymer matrix has an influence on the failure of the laminate. Under compressive loading along the fiber direction, shear strains are generated in the matrix due to excessive rotation of fibers causing the matrix to damage, and vice-versa. The laminate fails in the form of kink band or micro-buckling [7-10]. Therefore, it is very critical to account for the damage occurring in the polymer matrix in order to determine the extent of damage accumulation in the fiber-reinforced PMC.

Therefore, the purpose of this paper is to model progressive damage and failure to determine the amount of damage associated with a unidirectional fiber-reinforced PMC under any load configuration. To predict the progressive damage of the PMC, this model is based on a thermodynamically-based work potential theory developed by Schapery[11]. Past publications have used crack density, geometry, strain energy release rate, and other crack models to predict damage evolution [12-14]. The progressive damage in PMC is accounted by matrix micro-damage, which yields to more severe failure mechanisms. Therefore, ST has been extended up to failure in order to account for the maximum strain and amount of damage at which the fiber-reinforced PMC completely fails.

2. Methodology

2.1 Progressive Damage

Failure initiation in the PMC is given by its critical strains, which are obtained by physical experiments of a small-scale PMC. Elastic properties of the fiber and matrix, as well as the fiber volume fraction and matrix Poisson's ratio are known variables of the model. The fibers are modeled as elastic transversely isotropic material, which properties are shown in Table. 1. Likewise, the polymer matrix is modeled as elastic-plastic isotropic material, therefore, the equivalent stress-strain response is nonlinear, as shown in Fig. 1. This non-linearity in the stress-strain response of the polymer matrix is the representation of micro-structural damages that manifest progressive damage in the composite, reducing its stiffness. In other words, progressive damage is represented as the region before the critical strain and it is accounted by matrix microdamage, i.e. microcracking,

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void growth, fissuring, shear banding and fiber-matrix debonding.

Table 1 Fiber Properties.

E_{11} (GPa)	$E_{22}=E_{33}$ (GPa)	$\upsilon_{12} = \upsilon_{13} = \upsilon_{23}$	$G_{12}=G_{13}$ (GPa)	G_{23} (GPa)
276	8.76	0.35	12.0	3.244

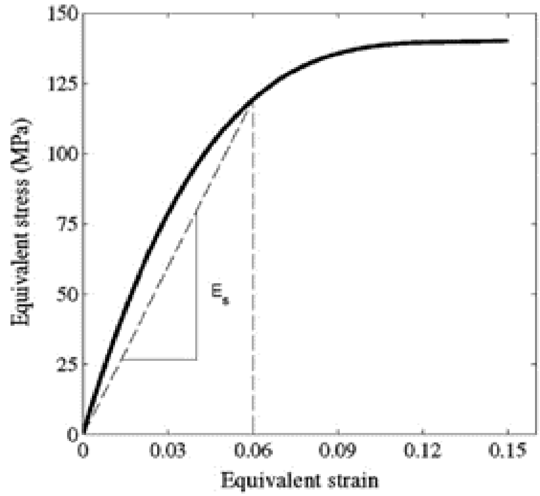


Fig.1. Equivalent stress-strain response of the polymer matrix.

According Pineda [3] the response of a composite in compression (axial or transverse) can be very different from that in tension. This means, it is easier for cracks to advance more in tension than in compression. Transverse cracks in compression progress under pure mode 2 and 3 or mixed mode conditions and there are other mechanisms associated to the failure of the PMC, which are fiber kinking and microbuckling. For this reason compressive loads are not taken into account in this model. As suggested by Sicking [15], it is assumed that the material is elastic and there is no plastic deformation upon unloading. However, plastic deformation can be incorporated to the model as stated by Schapery [11]. The concentric cylinder model (CCM) is used to determine the upscaled PMC mechanical properties, utilizing only the basic constituent (fiber and matrix) properties, as explained in the article by Prabhakar and Waas[16].

The CCM equations corresponding to the elastic regime are extended into the inelastic regime, to homogenize the lamina beyond the elastic regime, using a series of values of secant moduli of the pure matrix material as opposed to a single value of elastic modulus. That is, `$E_m$' of the matrix is not a single value, but a series of values `E_s', where `$E_s$' is the secant modulus of the pure matrix as shown in Fig. 1. By substituting a series of values of `Es' in expressions for E_{11}, E_{22}, G_{12}, G_{23} obtained from the CCM, we obtain the corresponding series of values for these constants as a function of stress (or strain), based on the assumption of stress based or strain based derivation of the CCM equations. Here, E_{11} (fiber dominated) is strain based, and E_{22}, G_{12} and G_{23} (matrix dominated) are stress based calculations. This implies that matrix dominated properties are expressed as a function of stress; while fiber dominated properties are expressed as a function of strain.

2.2 Schapery Theory

Schapery Theory (ST) accounts for the progressive damage in the matrix of the PMC. According to ST, the total work potential, W_T, is equal to the sum of the recoverable work potential (elastic region), *W*, and the dissipated (irrecoverable) energy, W_S [11].

$$W_T = W + W_S \tag{1}$$

Schapery (1989, 1990) shows that:

$$\frac{\partial W_T}{\partial S_m} = 0 \tag{2}$$

In general, S_m accounts for any damage that a composite may experience. Therefore, W_s will be only function of a single value of S (W_s=S). Thus, equation 1 yields:

$$W_T = W + S \tag{3}$$

Since every time a composite material is loaded it undergoes structural changes (damage), thus, the mechanical properties of the fiber-reinforced PMC are affected. Likewise, the recovered energy is obtained once the material is unloaded and it follows the elastic path. In other words, dissipated energy S is shown to be the area above the unloaded line, while the elastic strain energy density W is the area below this line (triangular shaded area), as shown in Fig. 2.

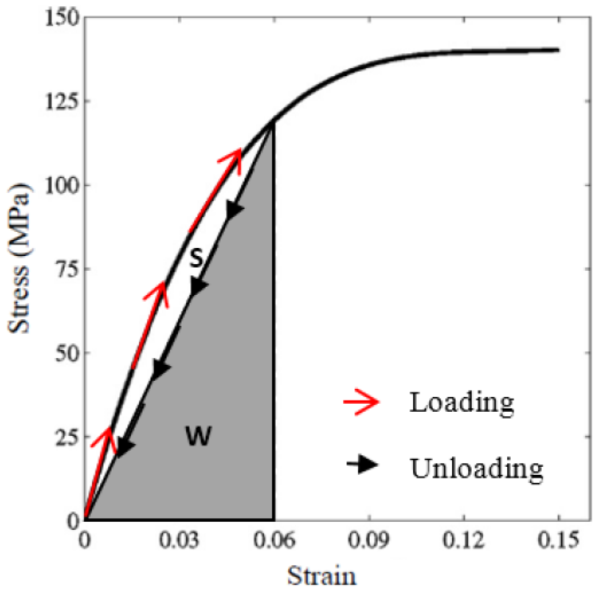


Fig.2. Stress-strain curve showing the elastic (*W*) and irrecoverable (*S*) portions divided by unloaded line.

Differentiating with respect to S, we have:

$$\frac{\partial W}{\partial S} = -1 \tag{4}$$

It is known that the amount of energy dissipated cannot be recovered, therefore:

$$S \geq 0 \tag{5}$$

Moreover, since the fiber-reinforced PMC is a unidirectional lamina, it is considered to be a transversely isotropic material and from the stress-stress relation $\{\sigma\} = [C]\{\varepsilon\}$ or simply $\sigma_i = C_{ij}\varepsilon_j$ for i, j =1,2,3…6, the elastic strain energy can be related to the elastic constantsC_{ij}.

$$W = \frac{1}{2}C_{11}\varepsilon_{11}^2 + C_{12}(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{11}\varepsilon_{33}) + \frac{1}{2}C_{22}(\varepsilon_{22}^2 + \varepsilon_{33}^2) + C_{23}(\varepsilon_{22}\varepsilon_{33}) + \frac{1}{2}C_{44}\varepsilon_{23}^2 + C_{55}(\varepsilon_{12}^2 + \varepsilon_{13}^2) \tag{6}$$

Where:

$$C_{11} = \frac{E_{11}^2(\nu_{23}-1)}{\Delta} \tag{7}$$

$$C_{12} = \frac{-E_{11}E_{22}\nu_{12}}{\Delta} \tag{8}$$

$$C_{22} = \frac{E_{22}(E_{22}\nu_{12}^2 - E_{11})}{(1+\nu_{23})\Delta} \tag{9}$$

$$C_{33} = \frac{-E_{22}(E_{22}\nu_{12}^2 - E_{11}\nu_{23})}{(1+\nu_{23})\Delta} \tag{10}$$

$$C_{44} = 2G_{23} \tag{11}$$

$$C_{55} = 2G_{12} \tag{12}$$

$$C_{66} = C_{55} \tag{13}$$

$$\Delta = 2E_{22}\nu_{12}^2 + E_{11}(\nu_{23} - 1) \tag{14}$$

Substituting Equation 6 into Equation 4, the derivative of the elastic strain energy with respect to the damage has to be equal to -1. It should be noted that now the C_{ij} are functions of the damage parameter (S). Therefore, we have,

$$\frac{\partial W}{\partial S} = \frac{1}{2}\frac{\partial C_{11}}{\partial S}\varepsilon_{11}^2 + \frac{\partial C_{12}}{\partial S}(\varepsilon_{11}\varepsilon_{22} + \varepsilon_{11}\varepsilon_{33}) + \frac{1}{2}\frac{\partial C_{22}}{\partial S}(\varepsilon_{22}^2 + \varepsilon_{33}^2) + \frac{\partial C_{23}}{\partial S}(\varepsilon_{22}\varepsilon_{33}) + \frac{1}{2}\frac{\partial C_{44}}{\partial S}\varepsilon_{23}^2 + \frac{\partial C_{55}}{\partial S}(\varepsilon_{12}^2 + \varepsilon_{13}^2) = -1 \tag{15}$$

Where, the critical strains of the fiber-reinforced PMC are constant and the microdamage functions (degrading elastic constants as a function of damage) are second order differential equations, so they can be solved for the damage evolution *S*.

2.3 Extended Schapery Theory

More catastrophic failure mechanisms due structural changes occur once the critical strain of the PMC is reached and the PMC stiffness decreases drastically until the PMC completely fails, as shown in Fig. 3. The critical strain is characterized as the damage initiation point, where the tangent stiffness tensor is not positive anymore and there is some damage located into the smallest length scale of the finite element volume (single element) [17]. Since the shaded area of the stress-strain response of a PMC (Fig. 3) describes the energy per unit volume dissipated during the failure process, the total amount of energy dissipated in the element tend to zero as the element length scale also approaches zero, leading to pathological dependence of the solution on the mesh density.

Under transverse tension and shear loading some transverse cracks might appear on the matrix. These transverse cracks are more severe than matrix microdamage because they are growing abruptly rather than progressive. Therefore, transverse cracking and fiber breakage are common failure mechanisms in fiber-reinforced PMC. The Crack Band model is used to manifest these failure mechanisms. This model relates the fracture toughness of the material to a characteristic finite element length scale, both are assumed to be constant throughout the model to overcome pathological mesh dependency. This means that the total strain energy release rate upon complete failure (shaded area in Fig. 3) is always equal to the fracture toughness of the material by scaling the stiffness, which is assumed to decrease linearly after the critical strain, to a characteristic length.

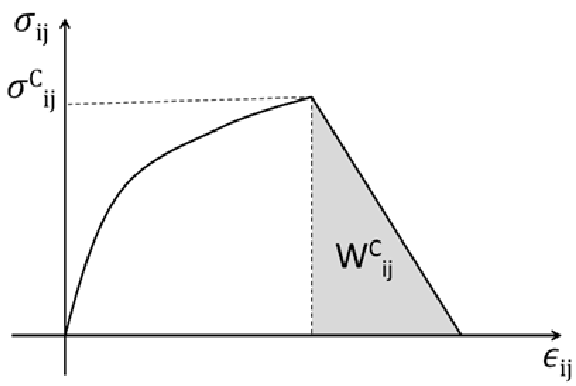
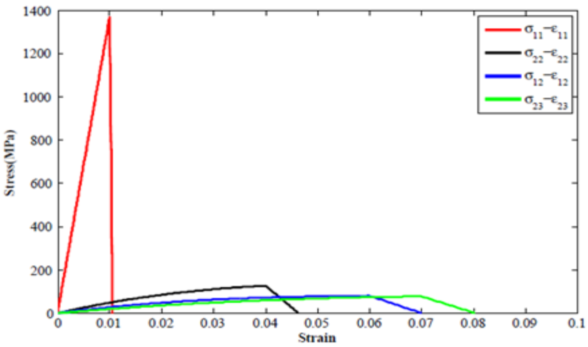


Fig.3 Schematic representation of stress-strain response of fiber-reinforced PMC up to complete failure.

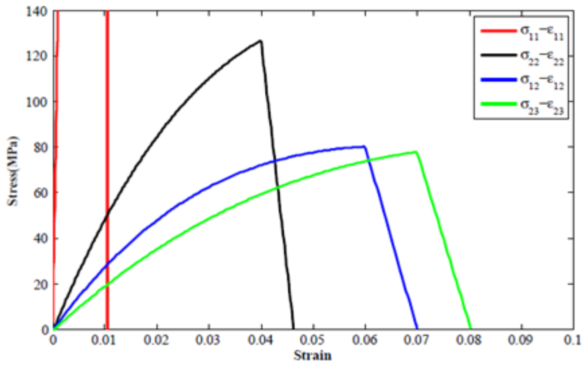
3. Results and Discussions

The stress–strain response for the homogenized lamina of PMC are obtained by the CCM model and the nonlinearity of E_{22}, G_{12} and G_{23} is verified and it is attributed to progressive damage of the polymer matrix. It is important to state that the axial response of the PMC is linear since it depends mainly on the axial stiffness of the fibers, which is very high compared to the stiffness of the polymer matrix. Otherwise, the transverse and shear axial responses are dominated by the behavior of the matrix, this is the reason of the non-linear stress-strain response. Then, the ST has been extended up to the failure state, at which the maximum strain supported by the PMC is reached and it completely fails, as seen in Fig. 4 (a) and (b). Since the stiffness in the 1 direction is much higher compared to the other directions, Fig. 4 (b) reproduces a better view of the non-linear stress-strain responses.

The elastic constants C_{ij} are plotted against the amount of damage (S), as shown in Fig. 5. black curves, while the fitted curves (second order polynomials) are colored. It can be observed in Fig. 5. (a) that the elastic constant in the 1 direction has much higher values than the other constants. Then, to better visualize these other constants, the plot is zoomed and shown in Fig. 5 (b). Similarly, Table 2, shows the results obtained for different configurations of strain loading, i.e. uniaxial, biaxial, multiaxial and combined axial/transverse-shear loading.

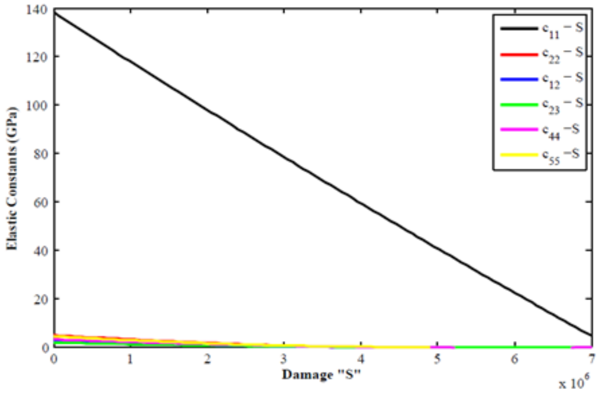


(a)

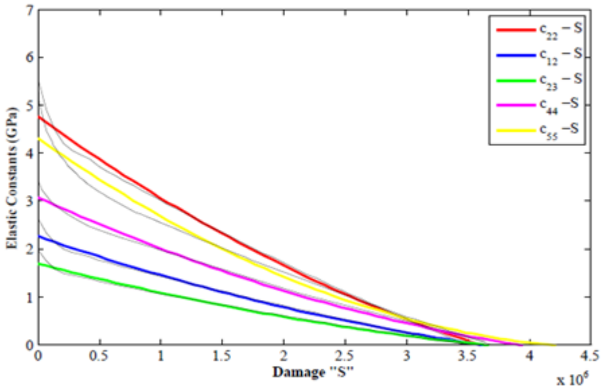


(b)

Fig.4 Stress-strain responses of a homogized lamina up to failure. (a) Complete representation of the responses (b) Zoomed plot to visualize nonlinearity.



(a)



(b)

Fig.5 Elastic constants (*Cij*) vs amount of damage (*S*). (a) Complete representation (b) Zoomed plot to visualize a comparison of original plots and approximated curve.

Table 2 Results obtained for amount of damage (*S*) for different configurations of strain applied.

ε_{11}	ε_{22}	ε_{33}	ε_{12}	ε_{23}	Amount of Damage
0.005					0
	0.03				1.43E3
	0.04				1.95E5
	0.05				2.80E5
	0.06				3.3E5
	0.06	0.03			3.56E5
0.05	0.06	0.03			3.89E5
				0.05	3.4E5
			0.03		3.86E5
	0.03		0.03		3.92E5
	0.03		0.03	0.05	3.95E5
			0.05		4.17E5

4. Conclusions

A thermodynamically-based work potential theory developed by Schapery was presented to account for the effects of progressive matrix microdamage in fiber-reinforced PMC. Shapery Theory was expended up to the failure state to capture more catastrophic failure mechanisms as transverse cracking and fiber breakage. This model was based in the CCM and Crack Band models, which were useful to determine the homogenized mechanical properties of the fiber-reinforced PMC and to relate the fracture toughness of the PMC with a finite element characteristic length scale, respectively. It is concluded that the amount of damage is bigger for multiaxial and combined axial/transverse-shear strain loading configuration. For uniaxial and biaxial strain, the amount of damage is small if this value is less than the critical strain of the PMC in the corresponding direction. However, damage increases drastically once the critical strain is reached. A finite element computational model can be developed to verify these results, which is currently being studied.

NOMENCLATURE

E_{ij} : Stiffness, GPa
ε_{ij} : Strain
υ_{ij} : Poisson's ratio
G_{ij} : Shear modulus, GPa
E_m : Elastic modulus of matrix, GPa
E_s : Secant modulus, GPa
W_T : Total work potential
W : Recoverable work potential
Ws : Dissipated energy
S : Damage parameter
Cij : Elastic constants

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