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Transportation Cost Optimization Using Linear Programming

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ABSTRACT
Optimization means using resources and existing technology at the best possible way. Better planning and its execution results in optimization of many problems. Quantitative models and mathematical tools such as linear programming allows for better result. We can use modern computing equipment for this purpose. Nowadays various problems of operational planning for transportation problems are solved by mathematical methods. Linear programming method is used to model most of these transportation problems. In this paper a real world application of a transportation problem that involves transporting mosquito coil from company's warehouse to distributor's warehouse is modeled using linear programming in order to find the optimal transportation cost. Excel Solver has been used to model and solve this problem.

Keywords: Optimization, Linear Programming, Transportation Cost, Supply Chain.

1. Introduction

To be successful in today's highly competitive marketplaces, companies must strive for greatest efficiency in all of their activities and completely utilize any possible opportunity to gain a competitive advantage over other firms. Among many possible activities, cost reduction in logistics is regarded as one of the core areas presenting enormous opportunities.

According to Jonsson there are two kinds of logistic costs: direct and indirect cost. Direct costs include physical handling, transportation, and storage of goods in the flow of materials together with the administration costs, whereas capacity and shortage costs are indirect costs. Jonsson also claims that direct logistics costs roughly vary between 10% and 30% of the turnover depending on the type of industry [1].

In such situation, it can be said that implementing optimization techniques to transportation of goods in order to schedule when and how much to send from each origin to its respective destination over a certain time period is a possible way to make improvements over the total cost of logistics.

In this paper a real world application of a transportation problem that involves transporting mosquito coil from company's warehouse to distributor's warehouse is modeled using linear programming in order to find the optimal transportation cost. Excel Solver has been used to model and solve this problem.

2. Linear Programming

Linear programming or linear optimization is a mathematical method for determining a way to achieve the best outcome (such as maximum profit or lowest cost) in a given mathematical model for some list of requirements represented as linear relationships. Linear programming is a specific case of mathematical programming (mathematical optimization) [2].

More formally, linear programming is a technique for the optimization of a linear objective function, subject to linear equality and linear inequality constraints.

Linear programs are problems that can be expressed in canonical form:

$$\begin{array}{ll} maximize & c^T x \\ subject\ to & Ax \leq b \\ and & x \geq 0 \end{array}$$

Where x represents the vector of variables (to be determined), c and b are vectors of (known) coefficients, A is a (known) matrix of coefficients, and $(.)^T$ is the matrix transpose. The expression to be maximized or minimized is called the objective function ($c^T x$ in this case). The inequalities $Ax \leq b$ are the constraints which specify a convex polytope over which the objective function is to be optimized. In this context, two vectors are comparable when they have the same dimensions. If every entry in the first is less-than or equal-to the corresponding entry in the second then we can say the first vector is less-than or equal-to the second vector [3].

Linear programming can be applied to various fields of study. It is used in business and economics, but can also be utilized for some engineering problems. Industries that use linear programming models include transportation, energy, telecommunications, and manufacturing. It has proved useful in modeling diverse types of problems in planning, routing, scheduling, assignment, and design [4].

3. Company Overview

J & J Essential Products (Pvt.) Ltd. is primarily a cosmetics and toiletries products manufacturing industry. However, they also produce some highly demanding products, i.e. electric bulb, mosquito coil etc. The cosmetics and toiletries products are branded as 'Jasmine' in the Bangladesh market place.

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2.1 About the Factory
The factory [5] is located at BSCIC industrial area, Golora, Manikganj, Bangladesh. It has one 5 storied building (main factory building) of 35000 sq. ft., each floor area covering 7000 sq. ft.

2.2 Marketing & Distribution of mosquito coil:
The Company has set up regional offices in Dhaka, Chittagong and Bogra along with warehouse facilities. There are seven distributor's warehouses where the goods are delivered from company's warehouses. The company sends goods through four of its own vehicles and use public transport services. The company's sales and distribution costs account for 19% of total cost.

4. Problem Statement
Since the optimization model that will be developed is expected to be applicable to different instances, this section starts with depicting the scope of the problem which is followed by an extended description of the problem through a case provided by the company.

The problem is to determine the optimal quantity of mosquito coil that should be delivered from company's each warehouse to different distributor's warehouse in order to obtain the minimum transportation cost.

The company delivers mosquito coils from its three warehouses in Dhaka, Chittagong and Bogra to seven distributor's warehouses in Barisal, Chittagong, Dhaka, Rajshahi, Rangpur, Sylhet and Khulna without considering the optimal quantity. So if the company applies linear programming to find the optimal quantity of mosquito coil to be delivered, it will be able to minimize the transportation cost significantly, which will result in increased profitability.

In order to determine the optimal quantity of mosquito coil that should be delivered from company's each warehouse to different distributor's warehouse for obtaining the minimum transportation cost, the following information were collected from the Supply Chain Director of the company:

Shipping cost: Average shipping costs of per carton mosquito coil from company's warehouse to different distributor's warehouse are given in the table below:

Table 1 Average shipping costs of per carton coil.

Distributor's warehouse / Company's warehouse	Dhaka	Chittagong	Bogra
Dhaka	15	160	100
Chittagong	160	12	260
Rangpur	154	315	56
Barisal	245	410	190
Rajshahi	130	290	58
Sylhet	125	427	204
Khulna	215	375	160

*All units are in Bangladesh Taka (BDT)

Storage capacity: Storage capacity of company's different warehouses:

Table 2 Average shipping costs of per carton coil.

Dhaka	3980
Chittagong	1785
Bogra	4856

*All units are in cartons

Demands: Average demand of different distributor's warehouses:

Table 3 Average shipping costs of per carton coil.

Dhaka	1168
Chittagong	1560
Rangpur	1439
Barisal	986
Rajshahi	1658
Sylhet	2035
Khulna	1159

*All units are in cartons

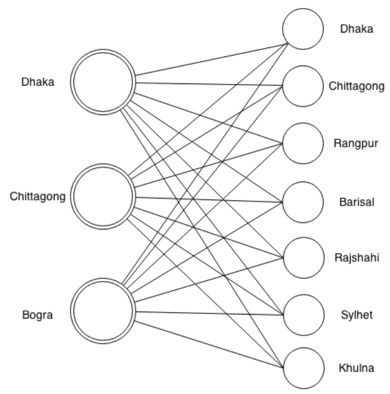


Fig.1 Illustration of the transportation network.

5. Mathematical Model Formulation

For the given problem, we formulate a mathematical description called a mathematical model to represent the situation. The model consists of following components:

Decision variables: These variables represent unknown quantities (number of items to produce, amounts of money to invest in and so on).

Objective function: The objective of the problem is expressed as a mathematical expression in decision variables. The objective may be maximizing the profit, minimizing the cost, distance, time, etc.

Constraints: The limitations or requirements of the problem are expressed as inequalities or equations in decision variables [6].

Followings are the decision variables, objective function and constraints specific to the problem of this paper:

Decision variables:

Three warehouses will be symbolized as:
Dhaka = A_1
Chittagong = A_2
Bogra = A_3

Seven distributor's warehouses will be symbolized as:
Dhaka = B_1
Chittagong = B_2
Rangpur = B_3
Barisal = B_4
Rajshahi = B_5
Sylhet = B_6
Khulna = B_7

Let the storage capacity of mosquito coils at A_i be a_i; where i=1, 2 and 3.

Let the requirements of mosquito coils at B_j be b_j; where j=1, 2, 3, 4, 5, 6 and 7.

Let C_{ij} be the cost of shipping one carton mosquito coil from A_i to B_j.

Let X_{ij} be the number of cartons of coil shipped from A_i to B_j.

Now let's assign the respective variables in Table 1:

Table 4 Average shipping costs with assigned variables.

Distributor's warehouse / Company's warehouse	Dhaka A_1	Chittagong A_2	Bogra A_3	Stock
Dhaka (B1)	$C_{2,1}$=15	$C_{2,1}$=160	$C_{3,1}$=100	b_1=1168
Chittagong (B2)	$C_{1,2}$=160	$C_{2,2}$=12	$C_{3,2}$=260	b_2=1560
Rangpur (B3)	$C_{1,3}$=154	$C_{2,3}$=315	$C_{3,3}$=56	b_3=1439
Barisal (B4)	$C_{1,4}$=245	$C_{2,4}$=410	$C_{3,4}$=190	b_4=986
Rajshahi (B5)	$C_{1,5}$=130	$C_{2,5}$=290	$C_{3,5}$=58	b_5=1658
Sylhet (B6)	$C_{1,6}$=125	$C_{2,6}$=427	$C_{3,6}$=204	b_6=2035
Khulna (B7)	$C_{1,7}$=215	$C_{2,7}$=375	$C_{3,7}$=160	b_7=1159
Requirement	a_1=3980	a_2=1785	a_3=4856	

Objective function:

The objective function contains costs associated with each of the variables. It is a minimization problem.

$$Minimize\ f = \sum_{i=1}^{3}\sum_{j=1}^{7} C\,(i,j)\,X(i,j)$$

Constraints:

The constraints are the conditions that force supply and demand needs to be satisfied. In the transportation problem, there is one constraint for each node.

The quantity of mosquito coils sent from A_i is $\sum_{j=1}^{7} X(i,j)$ and since the quantity of mosquito coils available at A_i is a_i, we must have $\sum_{j=1}^{7} X(i,j) \le a(i), where\ i = 1, 2\ and\ 3.$

Now, the quantity of mosquito coils sent to B_j is $\sum_{i=1}^{3} X(i,j)$ and since the quantity of mosquito coils required at B_j is b_j, we must have $\sum_{i=1}^{3} X(i,j) \ge b(j), where\ j = 1, 2, 4, 5, 6\ and\ 7.$

It is assumed that we cannot send a negative quantity from A_i to B_j, so $X_{ij} \ge 0$ for all values of i and j.

6. Modeling the Problem using Excel Solver

This section will demonstrate, how to use Excel Solver to find the optimum transportation cost.

The first step is to organize the spreadsheet to represent the model. Once the model is implemented in a spreadsheet, next step is to use the Solver to find the solution. In the Solver, we need to identify the locations (cells) of objective function, decision variables, nature of the objective function (maximize/minimize) and constraints.

Step by step solution of the problem using Excel Solver is given below:

Step 1:
At first we will construct a table in excel that will contain the cost parameters between each destination.

Shipping Costs

	Dhaka	Chittagong	Rangpur	Barisal	Rajshahi	Sylhet	Khulna
Dhaka	15	160	154	245	130	125	215
Chittagong	160	12	315	410	290	427	375
Bogra	100	260	56	190	58	204	160

Step 2:
Now we will construct another table that will contain shipment, stock and requirements.

Shipments

	Dhaka	Chittagong	Rangpur	Barisal	Rajshahi	Sylhet	Khulna	Total Out	≤	Stock
Dhaka	0	0	0	0	0	0	0	0	≤	3980
Chittagong	0	0	0	0	0	0	0	0	≤	1785
Bogra	0	0	0	0	0	0	0	0	≤	4856
Total In	0	0	0	0	0	0	0			
	=	=	=	=	=	=	=			
Requirement	1168	1560	1439	986	1658	2035	1159			

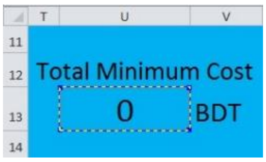
Here "Total In" is the quantity of coils shipped to that particular distributor's warehouse from company's three warehouses.

i.e. for Dhaka it is "=SUM(I7:I9)"

And "Total Out" is the quantity of coils shipped from that particular warehouse to distributor's seven warehouses.

i.e. for Dhaka it is "=SUM(I7:O7)"

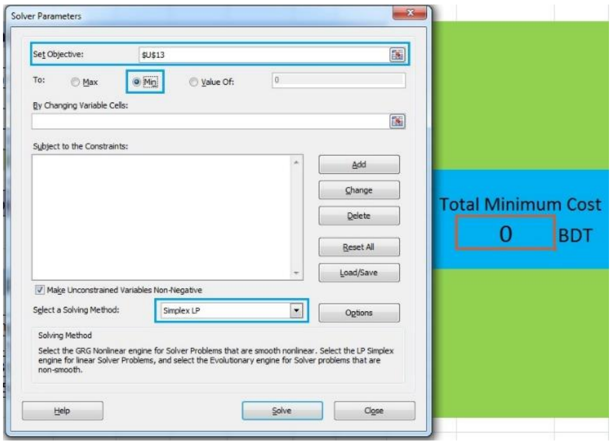
Step 3:
Now we will create a cell that will automatically calculate total cost based on the inputs in shipment table.



To calculate total cost we need the function SUMPRODUCT. This function will automatically sum all the product of unit and cost per unit.

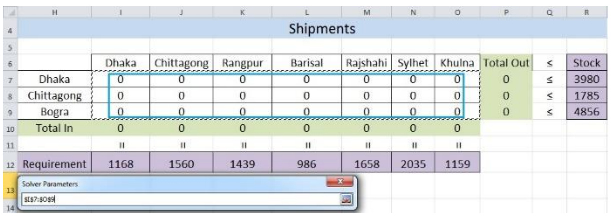
i.e. Total Cost =SUMPRODUCT(J18:P20,I7:O9)

Step 4:
In this step we will setup Excel Solver according to this problem. First we will open Solver from Data tab and locate the Total Minimum Cost cell in to 'Set Objective' in solver (i.e. $U$13). As we want to minimize the function we will choose 'Min'. Also we will choose our solving method as 'Simplex LP'.



Step 5:
In this step we will locate the changing variables (i.e. the optimal quantities) in to 'By Changing Variable Cells:' option of Solver.

Here changing cells will be $I$7:$O$9

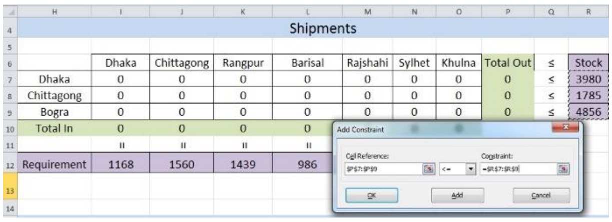


Step 6:
Now we will add constrains of this problem in Solver. As previously discussed, there are three constrains in this problem:

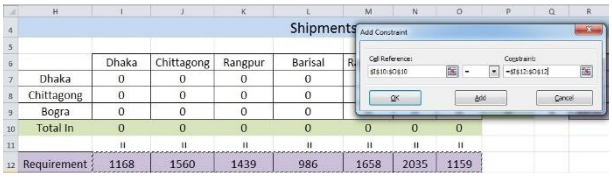
(1) Total Out is less than or equal to Stock
(2) Total In is equal to Requirement
(3) Shipment quantities are non-negative

To add constrains we will click add button on Solver and locate each constrains.

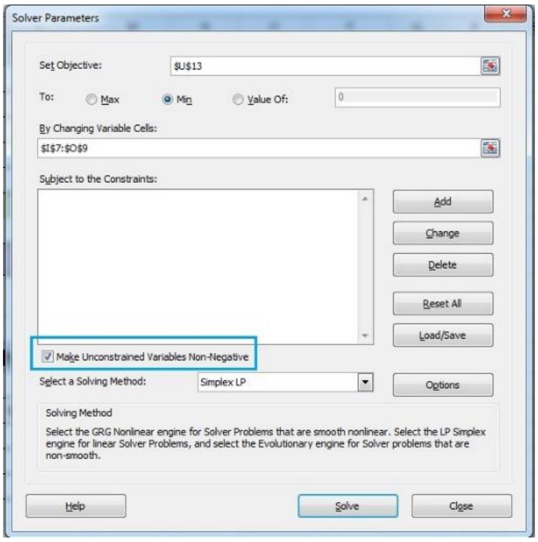
(1) Total Out is less than or equal to Stock ($P$7:$P$9 ≤ $R$7:$R$9)



(2) Total In is equal to Requirement ($I$10:$O$10 = $I$12:$O$12)

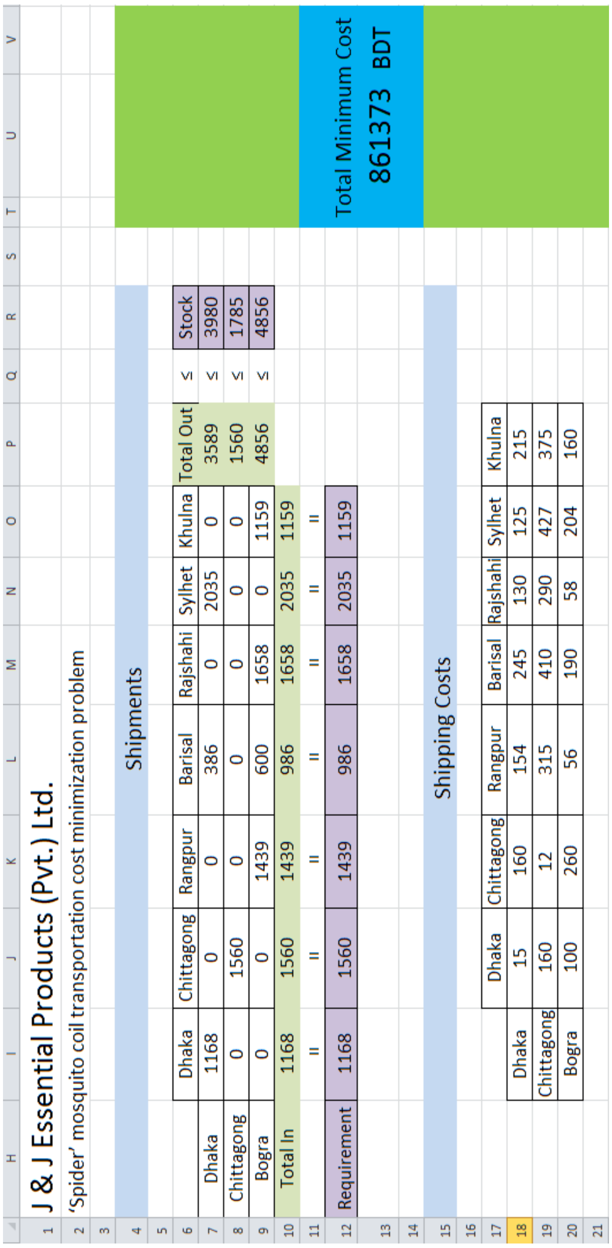


(3) Shipment quantities are non-negative (we will just select 'Make Unconstrained Variables Nonnegative')



Step 7:

Now we are done with the setup, all that is left is to click on ‘Solve’ button. This way we will find the following solution from Solver:



J & J Essential Products (Pvt.) Ltd.

‘Spider’ mosquito coil transportation cost minimization problem

Shipments

	Dhaka	Chittagong	Rangpur	Barisal	Rajshahi	Sylhet	Khulna	Total Out		Stock
Dhaka	1168	0	0	386	0	2035	0	3589	≤	3980
Chittagong	0	1560	0	0	0	0	0	1560	≤	1785
Bogra	0	0	1439	600	1658	0	1159	4856	≤	4856
Total In	1168	1560	1439	986	1658	2035	1159			
	=	=	=	=	=	=	=			
Requirement	1168	1560	1439	986	1658	2035	1159			

Total Minimum Cost

861373 BDT

Shipping Costs

	Dhaka	Chittagong	Rangpur	Barisal	Rajshahi	Sylhet	Khulna
Dhaka	15	160	154	245	130	125	215
Chittagong	160	12	315	410	290	427	375
Bogra	100	260	56	190	58	204	160

7. Limitations

In this problem we considered that supply will always exceed the demand. But in reality shortage of supply can exist.

Another importation limitation is that the company allocated 150 taka per carton for transportation cost when setting the trade price of their coil. So if the company makes a shipment that costs more than 150 taka per carton, it may incur loss.

7. Conclusion

Though this model has some limitations, still if the company applies the solution reached through this study, it will be able to minimize the transportation cost, which will result in increased profitability. In future a more optimized model of this problem can be developed by getting rid of the limitations mentioned above.

But again, certainly there will be some real life constrains, which we won’t be able to solve with any model. In those cases we will have to depend on our intuition and experience.

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