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Similarity Analysis of Unsteady MHD Boundary Layer Flow of Heat and Mass Transfer about an Inclined Stretching Porous Sheet

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ABSTRACT
The aim of the study is to investigate the similarity solution of unsteady MHD boundary layer flow of an incompressible, electrically conducting and viscous fluid about an inclined stretching sheet with suction by Quasi-linearization technique. So the present work is focused of the impact of the flow parameters on the velocity, temperature and concentration are computed, discussed and have been graphically represented in figures and also the shearing stress and rate of heat transfer shown in table 1 for various values of different parameters. In this regard the governing momentum boundary layer, thermal boundary layer and concentration boundary layer equations with the boundary conditions are transformed into a system of first order ordinary differential equations which are then solved numerically by using Runge-Kutta fourth-fifth order method along with shooting iteration technique. The results presented graphically illustrate that velocity field decrease due to increase of Magnetic parameter, porosity parameter, suction parameter and angle of inclination of the sheet and reverse trend arises for the increasing values of stretching parameter, unsteadiness parameter, Grashof number and modified Grashof number. The temperature field decreases for Magnetic parameter, porosity parameter, suction parameter and Prandtl number but the temperature field increases for the increasing values of stretching parameter, unsteadiness parameter, Grashof number and modified Grashof number. Again, concentration profile decreases for increasing the values of Magnetic parameter, porosity parameter, suction parameter and Schmidt number but concentration increases for increasing the values of stretching parameter, unsteadiness parameter, Grashof number and modified Grashof number. The present results in this paper are in good agreement with the work of the previous author.

Keywords: MHD; stretching sheet: angle of inclination; viscosity; porosity.

1. Introduction

The important applications of heat and mass transfer over a stretching sheet in spinning of fibers, extrusion of plastic sheets, polymer, cooling of elastic sheets etc. As a result the quality of final product depends on the rate of heat transfer so that the cooling procedure has to be maintained effectively. On the other hand, the MHD boundary layer flow of heat and mass transfer has significant applications in industrial manufacturing processes such as Magneto-hydrodynamics power generator, glass fiber production, plasma studies, cooling of Nuclear reactors, petroleum industries, and paper production etc. In this regard many investigators have studied the boundary layer flow of electrically conducting fluid due to stretching sheet in presence of magnetic field. For this reason, Elbashbeshy and Bazid [1] discussed the similarity solution for unsteady momentum and heat transfer flow whose motion is caused solely by the linear stretching of a horizontal stretching surface, Alharbi et.al [2] studied heat and mass transfer in MHD visco-elastic fluid flow through a porous medium over a stretching sheet with chemical reaction, Seddeek and Abdel Meguid [3] analyzed the effects of radiation and thermal diffusivity on heat transfer over a stretching surface with variable heat flux , Ali et al. [4] studied the Radiation and thermal diffusion effects on a steady MHD free convection heat and mass transfer flow past an inclined stretching sheet with Hall current and heat generation, Ibrahim and Shanker [5] investigated the unsteady MHD boundary layer flow and heat transfer due to stretching sheet in the presence of heat source or sink by Quasi-linearization technique. Ishak et al. [6] investigated the solution to unsteady mixed convection boundary layer flow and heat transfer due to a stretching vertical surface. Further, Ishak [7] studied unsteady laminar MHD flow and heat transfer due to continuously stretching plate immersed in an electrically conducting fluid. Ebashbeshy and Aldawody [8] analyzed heat transfer over an unsteady stretching surface with variable heat flux in presence of heat source or sink, Fadzilah et al. [9] studied the steady MHD boundary layer flow and heat transfer of a viscous and electrically conducting fluid over a stretching sheet with an induced magnetic field. Also, Bachok et al. [10] analyzed the similarity solution of the unsteady laminar boundary of an incompressible micro-polar fluid and heat transfer due to a stretching sheet and Mohebujjaman et al. [11] studied MHD heat transfer mixed convection flow along a vertical stretching sheet with heat generation using shooting technique. All of the above researchers in their studies were not consider the inclination of angle of the sheet and porosity. So the present work is focused on unsteady MHD boundary

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layer flow of an incompressible, electrically conducting and viscous fluid about an inclined stretching sheet with suction by Quasi-linearization technique.

2. Governing Equations of the Present Problem and Similarity Analysis

Let us Consider a two dimensional unsteady laminar MHD viscous incompressible electrically conducting fluid along an inclined stretching sheet with an acute angle (α) , X- axis is taken along the leading edge of the inclined stretching sheet and Y is normal to it. Also, a magnetic field of strength B_0 is introduced to the normal to the direction of the flow. Again, suppose that the uniform plate temperature T_w is grater than that of fluid temperature (T_∞), where T_∞ is the ambient temperature of the fluid. Let u and v be the velocity components along the X and Y axis respectively in the boundary layer region. Under the above assumptions and usual boundary layer approximation, the dimensional governing equations of continuity, momentum, concentration and energy under the influence of externally imposed magnetic field are:

Equation of continuity:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

Momentum equation:

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+ g\beta(T-T_\infty)\cos\alpha+g\beta^*(C-C_\infty)\cos\alpha -\frac{\sigma B_0^2 u}{\rho}-\frac{\nu}{k^*}u \tag{2}$$

Energy Equation:

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2} \tag{3}$$

Concentration Equation:

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D_m\frac{\partial^2 C}{\partial y^2} \tag{4}$$

Using free stream velocity $u=U(x,t)=\frac{bx}{\sqrt{1-\gamma t}}$,

we get

$$\frac{\partial U}{\partial t}+U\frac{\partial U}{\partial x}=-\frac{1}{\rho}\frac{\partial p}{\partial x}$$

Hence equation (2) becomes

$$\frac{\partial u}{\partial t}-\frac{\partial U}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=U\frac{\partial U}{\partial x}+\nu\frac{\partial^2 u}{\partial y^2}+ g\beta(T-T_\infty)\cos\alpha+g\beta^*(C-C_\infty)\cos\alpha-\frac{\sigma B_0^2 u}{\rho}-\frac{\nu}{k^*}u \tag{5}$$

The above equations are subject to the following boundary conditions:

$$\begin{aligned} &u=u_w,\ v=v_0(t),\ T=T_w, C=C_w \quad at\ \ y=0\\ &u=U, T=T_\infty, C=C_\infty \quad as\ \ y\to\infty \end{aligned} \tag{6}$$

The velocity of the sheet $u_w(x,t)$, the surface temperature of the sheet $T_w(x,t)$, concentration $C_w(x,t)$, and the transverse magnetic field strength $B(t)$ are respectively defined as follows:

$$u_w=\frac{ax}{\sqrt{1-\gamma t}}, T_w-T_\infty=\frac{bx}{\sqrt{1-\gamma t}}, C_w-C_\infty=\frac{cx}{\sqrt{1-\gamma t}},$$

$$B(t)=\frac{B_0}{\sqrt{1-\gamma t}}$$

where, a is the stretching rate and b, c are positive constant with dimension (time)$^{-1}$. We introduce the steam function $\psi(x,y)$ as defined by $u=\frac{\partial\psi}{\partial y}$ and $v=-\frac{\partial\psi}{\partial x}$.

To convert the governing equations into a set of similarity equations, we introduce the following similarity transformation:

$$\psi=x\sqrt{\frac{a\nu}{1-\gamma t}}f(\eta), \eta=y\sqrt{\frac{a}{\nu(1-\gamma t)}},$$

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \varphi(\eta)=\frac{C-C_\infty}{C_w-C_\infty}$$

From the above transformations, the non-dimensional, nonlinear and coupled ordinary differential equations are obtained as follows:

$$f'''+ff''-f'^2-A\left(f'+\frac{1}{2}f''\right)-(M+N)f' +Gr\theta\cos\alpha+Gm\varphi\cos\alpha+\frac{1}{2}A\lambda+\lambda^2=0 \tag{7}$$

$$\theta''+\Pr f\theta'-\frac{1}{2}\Pr A\eta\theta'=0 \tag{8}$$

$$\varphi'' + Scf\varphi' - \frac{1}{2}ASc\eta\varphi' = 0 \quad (9)$$

The transform boundary conditions:

$$f = -f_w,\ f' = 1, \theta = \varphi = 1\ at\ \eta = 0,$$
$$f' = \lambda, \theta = \varphi = 0\ \ as\ \eta \to \infty \quad (10)$$

Where f', θ and φ are the dimensionless velocity, temperature and concentration respectively, η is the similarity variable, the prime denotes differentiation with respect to η. Also

$$M = \frac{\sigma B_0^2(1-\gamma t)}{\rho a}, N = \frac{\nu(1-\gamma t)}{k^* a}, A = \frac{\gamma}{a}, \lambda = \frac{b\sqrt{(1-\gamma t)}}{a},$$

$$Gr = \frac{g\beta(T_w - T_\infty)(1-\gamma t)^2}{a^2 x}, Gm = \frac{g\beta^*(C_w - C_\infty)(1-\gamma t)^2}{a^2 x},$$

$$\Pr = \frac{\mu c_p}{k}, and\ \ Sc = \frac{\nu}{D_m}$$

are the Magnetic parameter, porosity parameter, unsteadiness parameter, stretching ratio, Grashof number, modified Grashof number, Prandtl number, and Schmidt number, respectively. The important physical quantities of this problem are skin friction coefficient C_f and the local Nusselt number Nu which are proportional to rate of velocity and rate of temperature respectively.

3. Methodology

The governing thermal boundary layer Eq. (3), concentration boundary layer Eq. (4) and momentum boundary layer Eq. (5), with the boundary conditions (6) are transformed into a system of first order ordinary differential equations which are then solved numerically by using Runge-Kutta fourth-fifth order method along with shooting iteration technique. From Eq. (7) – Eq.(9) it is observed that *f* is in third order and θ and φ are in second order. In order to solve this system of equations using Runge-Kutta method, the solution needs seven initial conditions but we have two initial conditions in f and one initial condition in each of θ and φ. The most important step of this scheme is to choose the appropriate finite value of η_∞. Therefore, to determine the value of η_∞, we have to start with some initial guess value and solve the boundary value problem consisting of Eq. (7)– Eq.(9). The solution process is repeated with another larger value of η_∞ until two successive values of $f''(0), \theta'(0)$ and $\varphi'(0)$ differ only after desired significant digit. The last value of η_∞ is taken as the finite value for determining the velocity, temperature and concentration, respectively. After getting all the initial conditions we solve this system of simultaneous equations using fourth order Runge-Kutta integration scheme. The effects of the flow parameters on the velocity, temperature and species concentration, are computed, discussed and have been graphically represented in figures and also the shearing stress and rate of heat transfer are shown in table1 for various value of different parameters. Now defining new variables by the equations

$$y_1 = f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta', y_6 = \varphi,$$
$$y_7 = \varphi'$$

The higher order differential Eq. (7), Eq. (8), Eq.(9) and boundary conditions (10) may be transformed to seven equivalent first order differential equations and boundary conditions are as follows:

$$dy_1 = y_2, dy_2 = y_3, dy_3 = -y_1 y_3 + y_1^2 + (M+N)y_2 +$$
$$A\left(y_2 + \frac{1}{2}y_3\right) - \frac{A\lambda}{2} - \lambda^2 - Gr\cos\alpha\, y_4 - Gm\cos\alpha\, y_6,$$

$$dy_4 = y_5, dy_5 = -\Pr y_1 y_5 + \frac{1}{2}\Pr A\eta\, y_5, dy_6 = y_7,$$

$$dy_7 = -Sc\, y_1 y_7 + \frac{1}{2}ScA\eta\, y_7$$

And the boundary conditions are

$$y_1 = -f_w, y_2 = 1, y_4 = 1, y_6 = 1\ \ at\ \eta = 0$$
$$y_2 = \lambda, y_4 = 0, y_6 = 0\ \ as\ \ \eta \to \infty$$

4. Results and discussion

Numerical calculation for distribution of the velocity, temperature and concentration profiles across the boundary layer for different values of the parameters are carried out. For the purpose of our simulation we have chosen f_w = 1.0, λ= 2.0, M = 0.2, N = 0.2, A = 1.0, Gr = 0.5, Gm=0.5, Sc= 0.22, Pr = 1.0 and α = 35° while the parameters are varied over range as shown in the figures. Fig.1 clearly demonstrates that the primary velocity starts from maximum value at the surface and then decreasing until it reaches to the minimum value at the end of the boundary layer for all the values M. It is interesting to note that the effect of magnetic field is more prominent at the point of peak value, because the presence of *M* in an electrically conducting fluid introduce a force like Lorentz force which acts against the flow if the magnetic field is applied in the normal direction as in the present problem. As a result velocity profile is decreased. Similar effect is also observed in Fig.5 and Fig.8 with increasing values of *α* and f_w. Fig.3, Fig.4, Fig.6 and Fig.7 show the velocity profile for various values of *λ, A, Gr* and *Gm*, it is observed that an increasing in *λ, A, Gr* and *Gm* lead to an increasing effect on velocity profile. From Fig. 2 it is observed that the velocity is decreased up to a certain interval of *η* and

then increased for increasing values of N. Fig.9 – Fig.16 show the temperature profile obtained by the numerical simulations for various values of entering parameters. Fig.13 clearly demonstrates that the thermal boundary layer thickness decreases as the Pr increases implying higher heat transfer. It is due to fact that smaller values Pr means increasing thermal conductivity and therefore it is able to diffuse away from the plate more quickly than higher values of *Pr*, hence the rate of heat transfer is reduced as a result the heat of the fluid in the boundary layer increases. Similar result has been found for the effect of *M*, *N* and f_w which are shown in Fig.9, Fig.10 and Fig.16 respectively and reverse trend arises for the increasing values of λ, *A*, *Gr* and Gm which are depicted in Fig.11, Fig.12, Fig.14 and Fig.15. From Fig. 9 it is observed that the temperature profile is decreased for increasing values of M which implies that the applied magnetic field normal to the flow of the fluid tends to reduce heat from the fluid and thus increases the rate of heat transfer as a result temperature is decreased. The effect of Gr on temperature profile is shown in Fig. 14. From this figure it is observed that, the temperature profile increases for increasing values of Gr; because the increase of Grashof number results in the increase of temperature gradients, which leads to the enhancement of the velocity due to the enhanced convection and thus temperature profiles are increased. Fig.17- Fig.24 shows the concentration profiles obtained by the numerical simulation for various values of entering non-dimensional parameters. From Fig.17, Fig.18, Fig.23 and Fig.24 it is observed that the concentration profile decreases for the effect of *M*, *N*, *Sc*, and f_w but reverse effect arises for the increasing values of λ, *A*, *Gr* and *Gm* which are shown in Fig.19, Fig.20, Fig.21 and Fig.22 respectively. Further the numerical solutions for the skin friction [$f''(0)$] and local Nusselt number [$-\theta'(0)$] have been compared with those of Pop et al. [12], Mahapatra and Gupta [13] and Sharma and Singh [14] when $M = 0$, $Gr = 0$, $Gm = 0$, $N = 0$, $A = 0$, $\alpha = 0$ and $f_w = 1.0$ and consider the Prandtl number $Pr = 0.01$. These results are given in Table.1 and it is observed that the agreement between the present results and those of Pop et al. [12], Mahapatra and Gupta [13] and Sharma and Singh [14] are familiar.

4. Conclusions

Following are the conclusions made from above analysis:

- The magnitude of velocity decreases with increasing magnetic parameter causing of Lorentz force reverse trend arise for stretching parameter and unsteadiness parameter.
- Increase in stretching parameter and unsteadiness parameter, the temperature is increased but reverse effect for Prandtl number.
- The noticeable increasing effects of stretching parameter, unsteadiness parameter, Grashof number and modified Grashof number but reverse trend arises for the increasing values of Schmidt number, magnetic parameter, porosity parameter and suction parameter on concentration profile.
- To compare the skin friction and rate of heat transfer with previous results and get almost similar results.

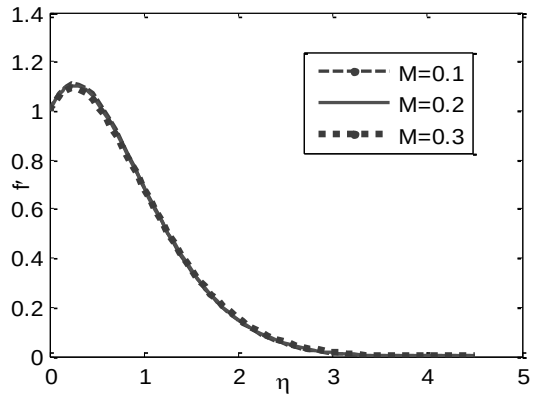

Fig.1 Velocity profile for various values of M

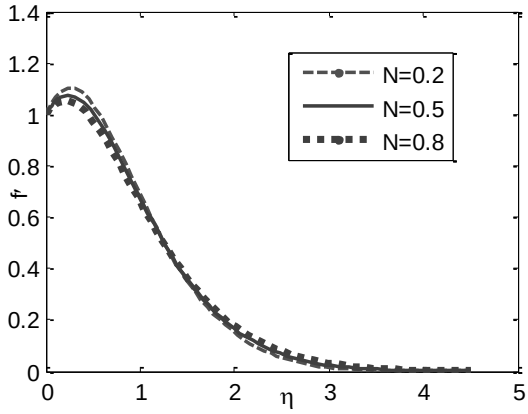

Fig.2 Velocity profile for various values of N

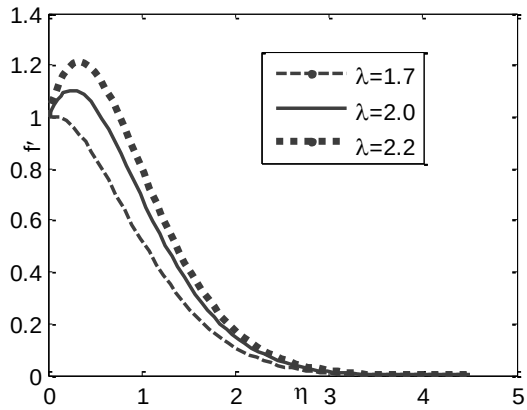

Fig.3 Velocity profile for various values of λ

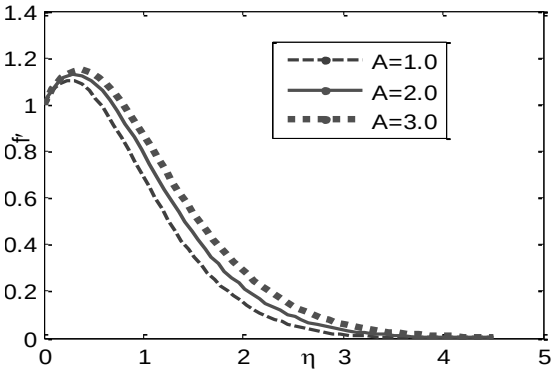

Fig.4 Velocity profile for various values of A

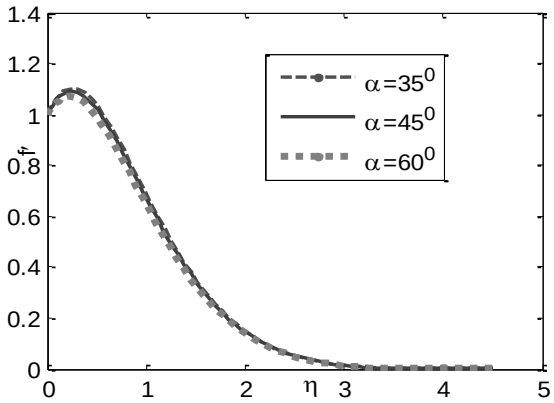

Fig.5 Velocity profile for various values of α

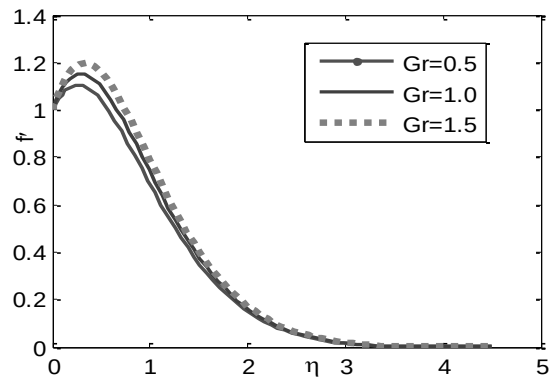

Fig.6 Velocity profile for various values of Gr

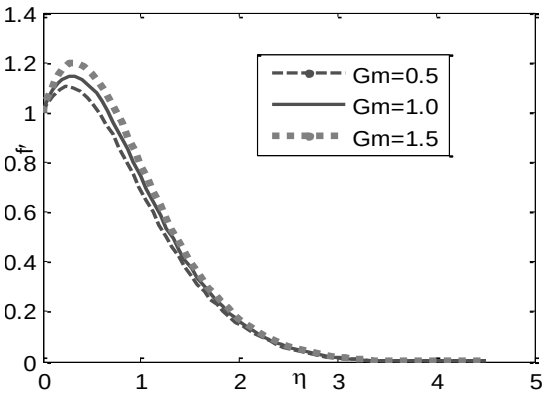

Fig.7 Velocity profile for various values of Gm

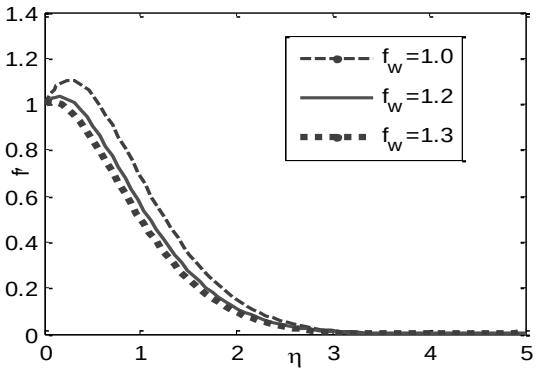

Fig.8 Velocity profile for various values of f_w

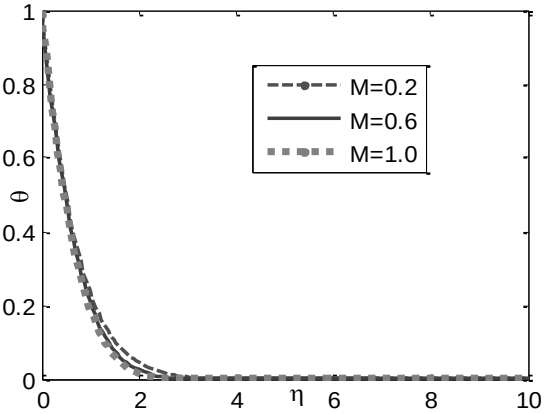

Fig.9 Temperature profile for various values of M

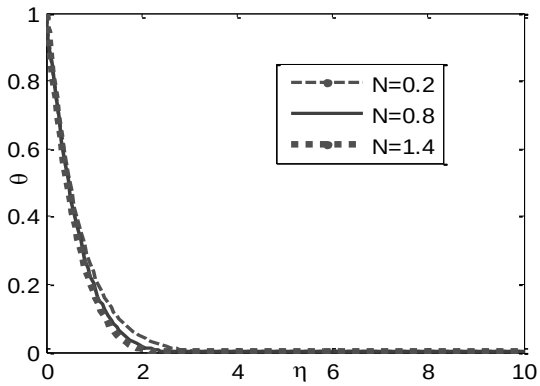

Fig.10 Temperature profile for various values of N

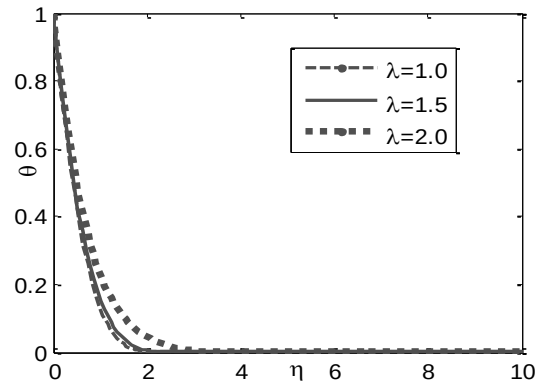

Fig.11 Temperature profile for various values of λ

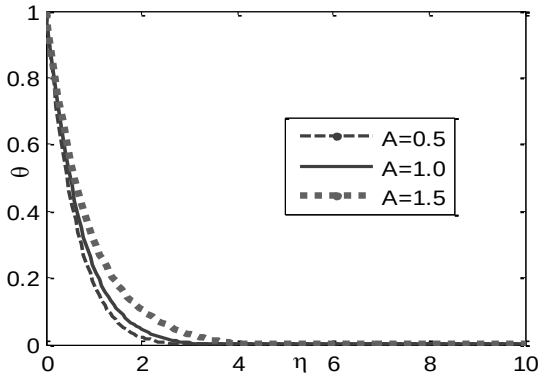

Fig.12 Temperature profile for various values of A

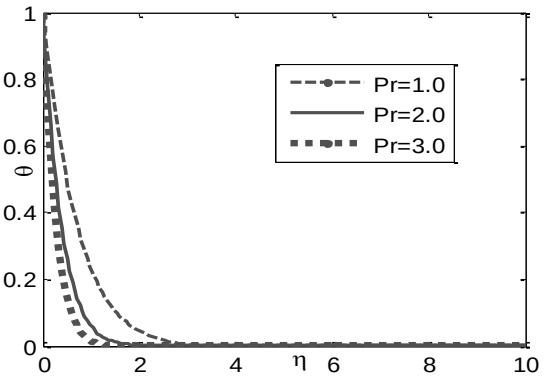

Fig.13 Temperature profile for various values of Pr

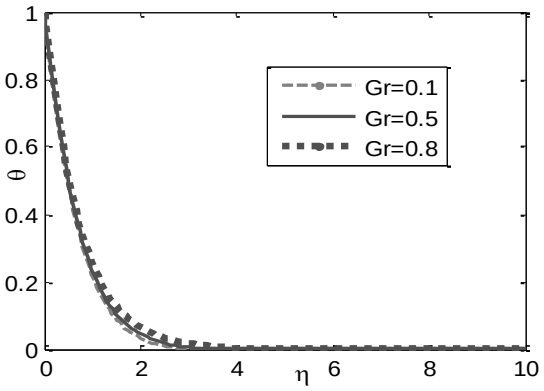

Fig.14 Temperature profile for various values of Gr

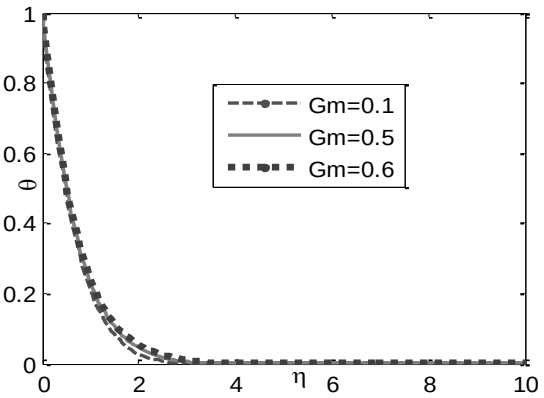

Fig.15 Temperature profile for various values of Gm

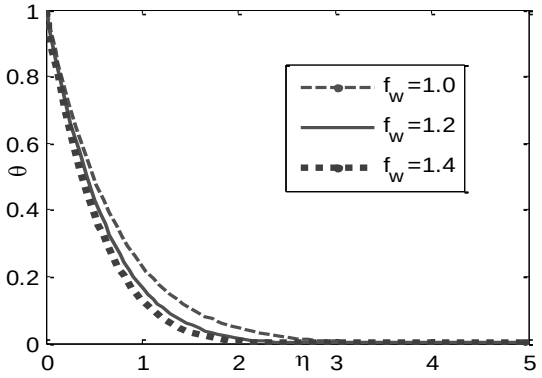

Fig.16 Temperature profile for various values of f_w

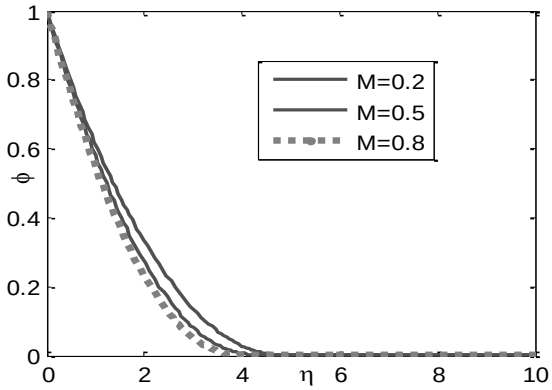

Fig.17 Concentration profile for various values of M

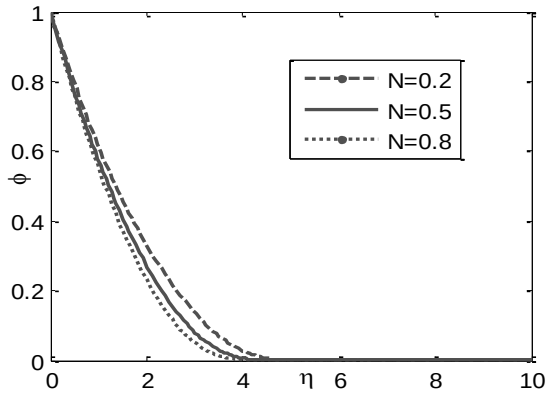

Fig.18 Concentration profile for various values of N

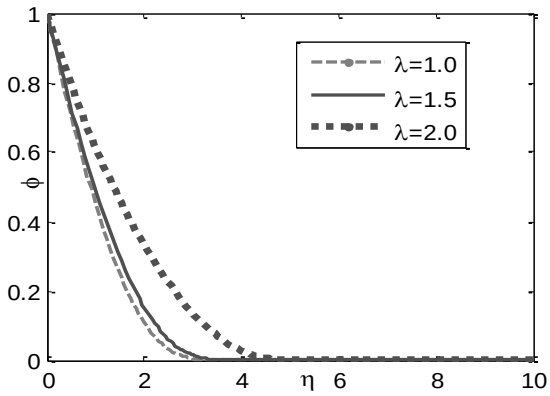

Fig.19 Concentration profile for various values of λ

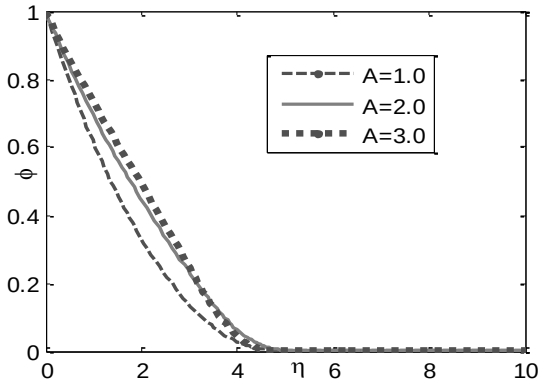

Fig.20 Concentration profile for various values of A

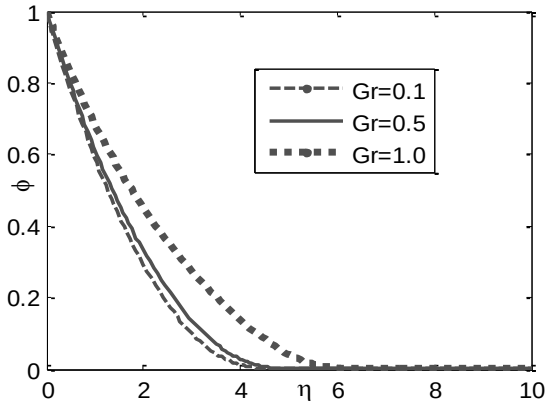

Fig.21 Concentration profile for various values of Gr

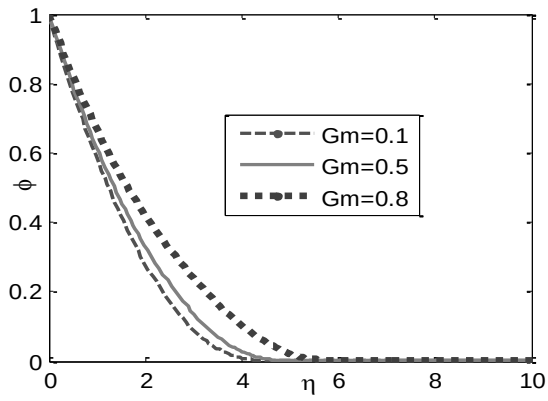

Fig.22 Concentration profile for various values of Gm

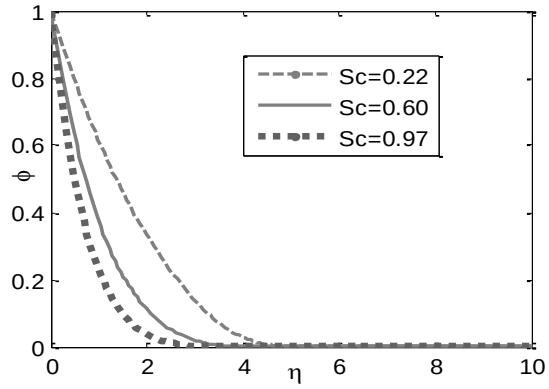

Fig.23 Concentration profile for various values of Sc

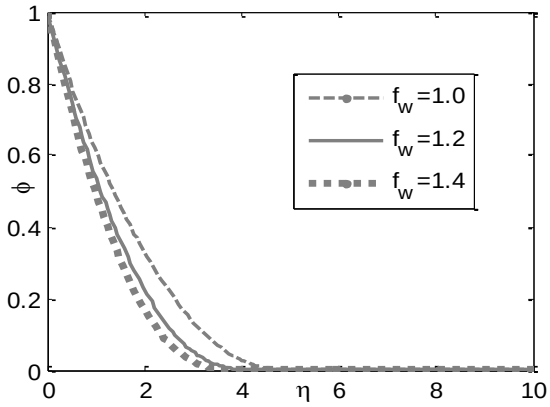

Fig.24 Concentration profile for various values of f_w

Table 1 Comparison of the skin friction [$f''(0)$] and local Nusselt number [$-\theta'(0)$]

λ	Pop et al.[12]		Mahapatra and Gupta[13]		Sharma and Singh [14]		Present results	
	$f''(0)$	$-\theta(0)$	$f''(0)$	$-\theta(0)$	$f''(0)$	$-\theta(0)$	$f''(0)$	$-\theta'(0)$
0.1	-0.9694	0.081	-0.9694	0.081	-0.969386	0.081245	-0.96782	0.099205
0.2	-0.9181	0.135	-0.9181	0.136	-0.9181069	0.135571	-0.911135	0.13539
0.5	-0.5573	0.241	-0.6673	0.241	-0.667263	0.241025	-0.7195	0.23926
2.0	2.0174	-	2.0175	-	2.01749079	-	1.9881	
3.0	4.7290	-	4.7293	-	4.72922695	-	4.72259	

NOMENCLATURE

c_p : specific heat at constant pressure, $Jkg^{-1}K^{-1}$
κ : thermal conductivity, $w\ m^{-1}K^{-1}$
α : angle of inclination, degree
g : acceleration due to gravity, ms^{-2}
γ : constant
σ : electrical conductivity, sm^{-1}
D_m : coefficient of mass duffisivity, m^2s^{-1}
μ : coefficient of viscosity, $kg\ m^{-1}s^{-1}$
ν :kinematics viscosity, m^2s^{-1}
ρ :fluid density, $kg\ m^{-3}$
B_0 :magnetic field intensity, Am^{-1}
β :thermal expansion coefficient, k^{-1}
β^* :concentration expansion coefficient, $\mu\ mm^{-1}k^{-1}$
u :velocity component along X axis, ms^{-1}
v :velocity component along Y axis, ms^{-1}
U :free stream velocity, constant
C :concentration, $kg\ m^{-3}$
C_w :stretching sheet concentration, $kg\ m^{-3}$
C_∞ :free stream concentration
T :fluid temperature, k^{-1}
T_w :stretching sheet temperature, k^{-1}
T_∞ :free stream temperature

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