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Numerical Analysis of Laminar Natural Convection from Heated Vertical Plate and Horizontal Cylinder

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ABSTRACT
The study aims to investigate the fluid flow and heat transfer characteristics around the vertical plate and horizontal cylinder. In this study, the constant temperature boundary condition is applied .The two dimensional mathematical models consisting the systems of partial differential equations such as Continuity equation, Navier-Stokes equation and Energy equation are used here. The solution was obtained using a commercial CFD code. Further grid independence test was undertaken to justify the value of numerical solutions. Results were basically obtained for a Prandtl number of 0.7; this being approximately the value of air. Numerical results are obtained in terms of Nusselt and Rayleigh Numbers. After comparing the numerically simulated figure and conventional figure found in literature, it is seen that the trend lines are similar for each and every cases. And finally, the numerical values are compared with the established empirical relation of Churchill and Chu, Merk and Prins, Morgan, McAdams etc.

Keywords: Natural convection, Heat Transfer, Numerical Analysis, Vertical Plate, Horizontal Cylinder.

1. Introduction
Natural convection heat transfer from a vertical plate and horizontal cylinder has been studied experimentally for more than fifty years. It is reported by the researchers that the obtained results show high levels of deviation among each other due to various reasons such as under sized test cabin, faulty temperature measurement system etc. This is why natural convective heat transfer from vertical plate and horizontal cylinder in laminar flow regions has been numerically studied to get better accuracy.
The problem of boundary layer free convection flow past a vertical plate and horizontal cylinder under different conditions was studied by many researchers. Several techniques and process of solution had been developed for the qualitative and quantitative analysis of convection, amongst which, one may make reference to experimental techniques, pure theoretical analysis and numerical simulation. Empirical approaches based on scaling analysis and finished by experimental information, and most recently, numerical simulations techniques, arisen to the computational advances, have been used in the study of natural convection.
Here heated objects are placed on the atmospheric condition and the flow phenomena are observed. Five different diameters are used for horizontal cylinder while a single plate is used for vertical plate. The GAMBIT mesh generator associated with FLUENT has been used to plot and mesh the 2D model of the vertical plate and cylinder. Grids are used according to the specific requirement for both cases. For cylinder, the grids are clustered close to the small cylinder while for grid adjacent to the wall is finer compared to that in the central region.
Natural Convection Heat Transfer from vertical plate involves in number of practical situations involving electrical and electronic equipment such as printed circuit boards, chips, conductors, solar cells, measurement systems etc. The size and inclination angle affect heat transfer from vertical plate and thus play a vital role in thermal design and manufacture of electronic equipment.
Free convective heat transfer from cylinders having circular and square cross sections has significant practical importance and is relevant to numbers of applications in the cooling of electrical and electronic components such as square pin fin heat sinks, resistors, capacitors, conductors, transformers, diodes, thyristors etc.

2. Physical Model of the Computational Domain
The height of the plate = 0.2 m

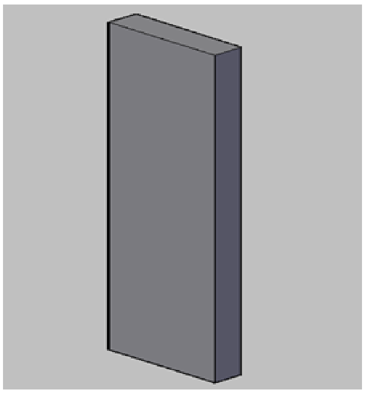

Fig.1 Vertical plate

The diameters of the cylinders are 0.05 m, 0.07 m, 0.1 m, 0.12 m and 0.15 m

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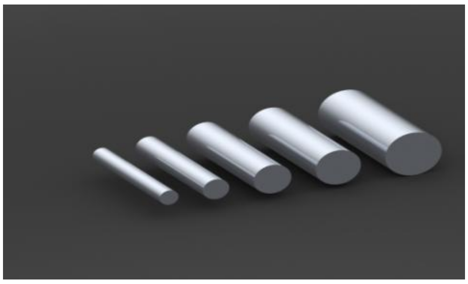

Fig.2 Horizontal Cylinder of various diameters

3. Boundary Conditions

The Physical boundary conditions for the problem include zero velocity components at the wall surface, no axial velocity outside the boundary layer, temperatures T_s at the wall and T_∞outside the boundary layer. These boundary conditions are stated as follows:

At y=0 (wall)
i)U=0
ii)V=0
iii) T=T_s
At y→∞ (or outside boundary layer)
i)U=0
ii) T=T_∞

Boundary conditions used for vertical plate and horizontal cylinder is shown graphically in the Figure 6 and Figure 7

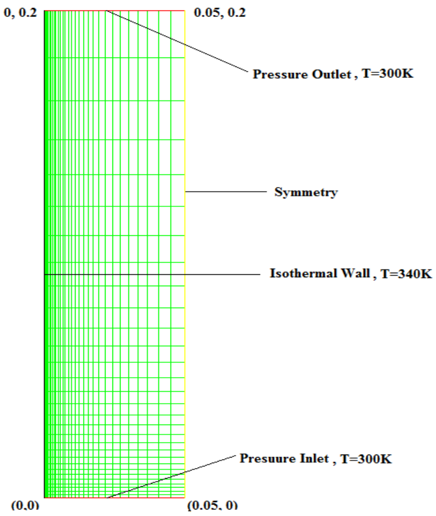

Fig.3 Computational Domain with boundary conditions for vertical plate

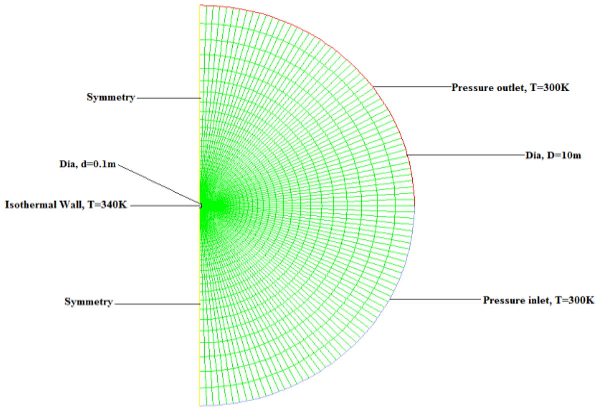

Fig.4 Computational Domain with boundary conditions for horizontal cylinder

5. Initial Conditions:

Material: Aluminum
Fluid: Air
Temperature of the wall, T_s = 340 k
Bulk temperature of the fluid, T_∞= =300 k

6. Solution Methods

Here the two problems are investigated using FLUENT as a 2D steady laminar flow problem. The heat transfer module of the FLUENT solver was enabled. The operating pressure of the domain was specified as 101325 Pa. At the heated wall, an isothermal boundary condition was used. At the bottom and top of the domain, a pressure inlet and outlet condition was specified. A symmetry boundary condition was imposed on the right end of the domain (Vertical plate) and on the left end of the domain (Horizontal Cylinder). Both the flow and energy equations were solved using the SIMPLE algorithm for pressure-velocity coupling. The PRESTO Scheme was used to discretize the pressure equations, and the momentum as well as the energy equations used a second order upwind scheme for spatial discretization. For this discretization, gradient is evaluated using a Least Squares Cell Based Method. The Under- Relaxation Factors used for pressure, density, body forces, momentum and energy is set to 0.3, 1, 1, 0.7 and 1 respectively. Boussinesq approximation was used for density evaluation. The Boussinesq approximation neglects the effect of fluid-density dependence on pressure of the air phase, but includes the density dependence on temperature. It enables the flow to be treated as incompressible flow but still accounts for density variation locally in the momentum and energy equations. As the Rayleigh number of the flow will be less than the critical value, no turbulence models were used in the solution. Here the convergence criteria for mass, momentums and energy equations have been set at 1E-6

7. Equations

Continuity equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

Navier- Stokes equation

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \vartheta\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right) \qquad (2)$$

Energy equation

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho c_p}\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right) \qquad (3)$$

Equations for Nusselt number, heat transfer co-efficient and boundary layer thickness calculation:

1. $Nu = Q_p {*} X_p$ or $D/(T-T_b){*}K$ [3]
2. $Nu_x = 0.508 * Ra_x^{1/4} * \left(\frac{Pr}{0.952+Pr}\right)^{1/4}$ [3]
3. $Ra_x = Gr_x * Pr$ [3]
4. $Nu_m = 0.68 + \frac{0.67 * Ra^{1/4}}{[1+(0.492/Pr)^{\frac{9}{16}}]^{4/9}}$ [8]
5. $h_x = 0.508 * Pr^{1/2} * \frac{Gr^{1/4}}{(0.952+Pr)^{1/4}} * \frac{k}{x}$ [1]
6. $\delta = 3.93 * X * [\frac{0.952+Pr}{Pr^2 * Gr}]^{1/4}$ [1]
7. $Nu_m^{1/2} = 0.60 + \frac{0.387 * Ra^{1/6}}{[1+(0.559/Pr)^{\frac{9}{16}}]^{8/27}}$ [5]
8. $Nu = C * Ra^{1/4}$ [4]
9. $Nu = C * Ra^n$ [7]

8. Results

After solving the models in FLUENT, the result from the numerical analysis is checked for mesh independence. Also the numerical result is validated using available experimental data. After successful validation, the result is post-processed using FLUENT for visualizing the flow behavior & heat transfer characteristics.

8.1 Comparison with Standard Results

For the test results to be accepted, the computed flow field should qualitatively agree with the general understanding of the flow physics expected for natural convection flows along heated vertical plate and horizontal cylinder. A comparison between conventional figure and figure from the result of numerical solution is done in order to validate the test results.

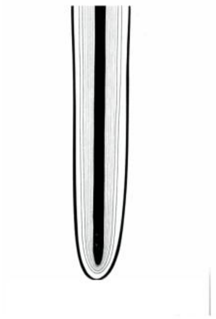

Fig.5 Interferometer photograph showing lines of constant temperatures around a heated vertical plate in natural convection [1]

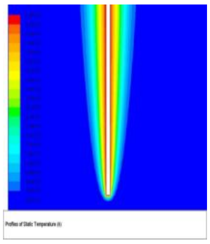

Fig.6 Color contours of constant temperatures around a heated vertical plate from the problem investigated

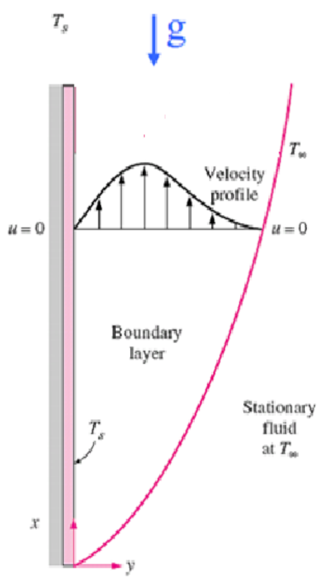

Fig.7 Velocity profiles for free convection from a hot vertical plate [1]

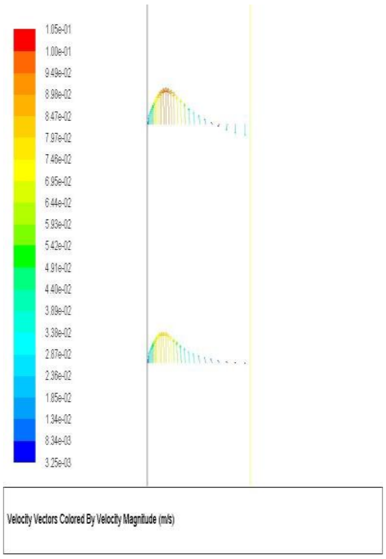

Fig.8 Velocity profiles for free convection from the hot vertical plate of the problem investigated

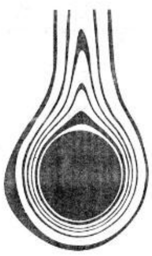

Fig.9 Interferometer photograph showing lines of constant temperatures around a heated horizontal cylinder in natural convection [1]

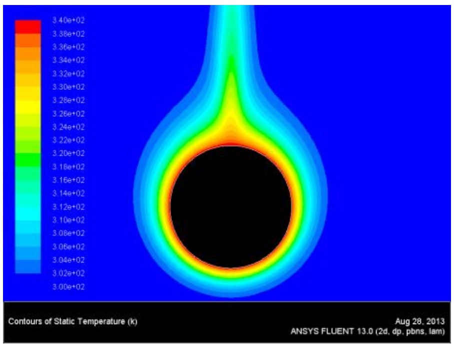

Fig.10 Color contours of constant temperatures around a heated horizontal cylinder (.1 dia) from the problem investigated

8.2 Validation of Numerical Result

In general, there is a higher degree of uncertainty between various experimental results. So it becomes troublesome to match the numerical values with the experimental value. Hence matching the numerical values within ±5 percent [2] could be considered sufficient acceptance condition.

For Vertical Plate:

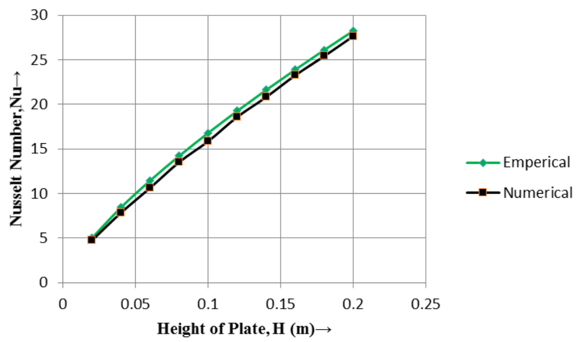

Fig.11 Comparison of Empirical and Numerical values of Nusselt Number along the vertical length of the heated plate

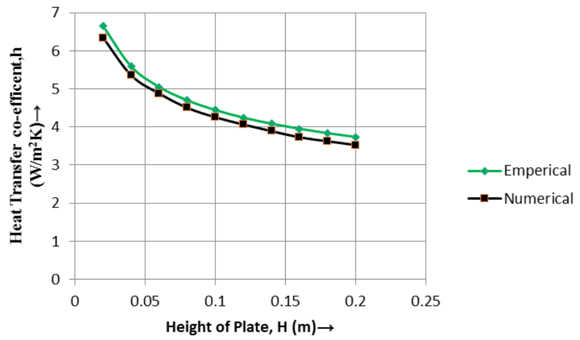

Fig.12 Comparison of Empirical and Numerical values of Heat Transfer Co-efficient along the vertical length of the heated plate

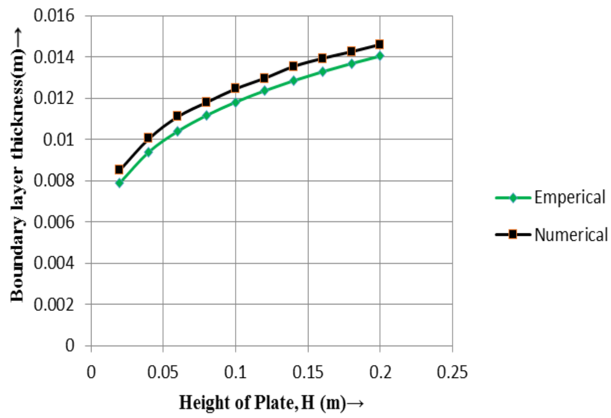

Fig.13 Comparison of Empirical and Numerical values of boundary layer thickness at various points on the heated vertical plate

For Horizontal Cylinder:
Nusselt number derived from various correlations is compared with the numerical value. From the figure 14, 15 and 16 it can be seen that the empirical and numerical values fall in ±5percent to ±7percent band respectively.

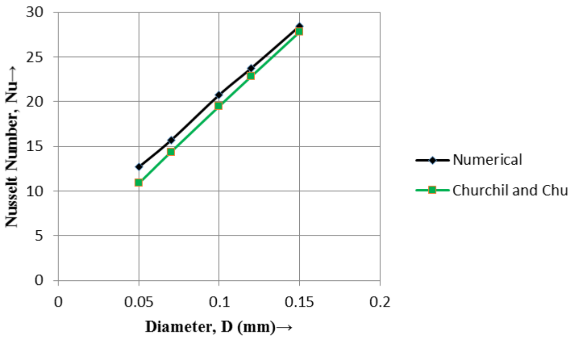

Fig.14 Comparison of Empirical [5] and Numerical values of Nusselt Number for heated horizontal cylinder of various diameters

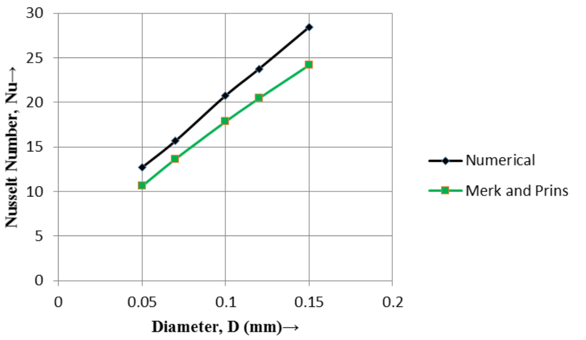

Fig.15 Comparison of Empirical [6] and Numerical values of Nusselt Number for heated horizontal cylinder of various diameters

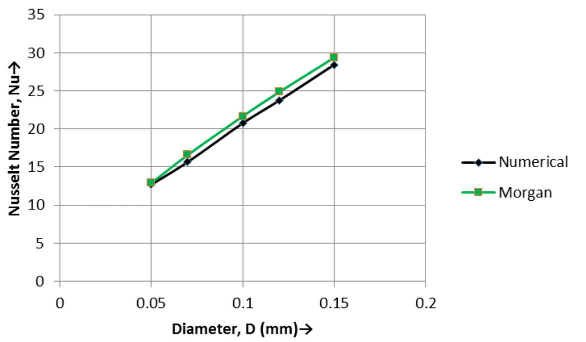

Fig.16 Comparison of Empirical [7] and Numerical values of Nusselt Number for heated horizontal cylinder of various diameters

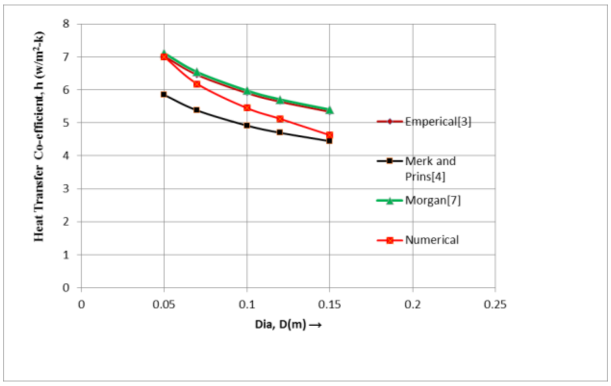

Fig.17 Comparison of Empirical [3], [4],[7] and Numerical values of heat transfer co-efficient for heated horizontal cylinder of various diameters

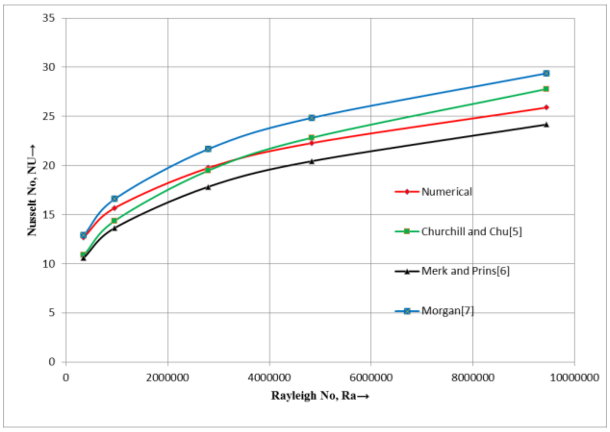

Fig.18 Comparison of Empirical [5], [6],[7] and Numerical values of Nusselt Number for various Rayleigh Number

9. Conclusion

1. For the vertical heated plate, both hydrodynamic and thermal boundary layer increases as the flow moves vertically upward. This is consistent with analytical solution [1].
2. The pattern of the boundary layers for heated horizontal cylinder satisfies the criteria of overall goodness and established understanding of the physical phenomena.
3. Comparison of numerical and analytical results for heat transfer coefficient for various diameter of horizontal cylinder was seen to fall into ±20% band.
4. Nusselt number increases with the increase of vertical length along the plate. So along the length of the vertical plate, convection becomes dominant than conduction.
5. Heat transfer co-efficient is inversely proportional to the distance along vertical plate while the boundary layer thickness increases with the increase of the distance.
6. Nusselt numbers obtained from these studies are compared with the correlations of Churchill and Chu [33], Merk and Prins [34], Morgan [35] according to the

different test cylinders. In addition to this, majority of data belong to other correlations were seen to fall into the 20% deviation line.
7. Nusselt number increases with increasing Rayleigh number in this study.

NOMENCLATURE:

*K***:** Thermal conductivity, $W \cdot m^{-1} \cdot K^{-1}$
*Nu***:** Nusselt number
*Ra***:** Rayleigh number
*Gr***:** Grashof number
*Pr***:** Prandtl number
C_{p}: Specific heat at constant pressure, $J \cdot kg^{-1} \cdot K^{-1}$
*T***:** Constant temperature of the walls, K
T_b : Fluid temperature, K
Q: Heat Flux, $W \cdot m^{-3}$
X: Location along the vertical plate, m
D: Diameter of the cylinder, m
x, *y***:** Cartesian co-ordinates
*ρ***:** Density of fluid, $Kg \cdot m^{-3}$
ϑ: Kinematic viscosity of fluid, $Kg \cdot m^{-1} \cdot s^{-1}$
ψ : Stream function, $Kg \cdot s^{-1}$
λ**:** Relaxation factor
μ: Dynamic viscosity of fluid, $Kg \cdot m^{-1} \cdot s^{-1}$
α: Thermal diffusivity, $m^{2} \cdot$ s-1
β Thermal expansion co-effiecient, $1 \cdot k^{-1}$
u, ***v*****:** Velocity components, $m \cdot s^{-1}$
*g***:** Gravitational acceleration, $m \cdot s^{-2}$
*h***:** Heat Transfer Co-efficient, $W \cdot m^{-2} \cdot k^{-1}$

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