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WEIGHTED COST OPPORTUNITY BASED ALGORITHM FOR INITIAL BASIC FEASIBLE SOLUTION: A NEW APPROACH IN TRANSPORTATION PROBLEM

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ABSTRACT

A large numbers of transportation algorithms are available in literature concerned with cost matrix, i.e., manipulation of cost entries to form Distribution Indicator (DI) or Total Opportunity Cost (TOC) Table. It is also observed that none of the works consider Supply and/or Demand for formulation of DI or TOC table. But it may be assumed that supply and demand entries play a vital role in the formulation of cost allocation table to obtain a better solution. In this article a weighted cost based TOC table is formulated by considering supply and demand entries as a weight factor. Then by incorporating Weighted Opportunity Cost table on the Least Cost Matrix method, a weighted cost opportunity based algorithm is proposedfor finding Initial Basic Feasible solution of Transportation Problem (TP). Some experiments have been carried out to justify the validity and the effectiveness of the proposed algorithm in which a new idea is incorporated.

Keywords:*Transportation Problem, Initial Basic Feasible Solution, Supply and Demand.*

1. INTRODUCTION

Transportation Problems (TP) playa very important role to ensure in time availability of raw materials and finished goods from different sources to distinct destinations. Only a strong network based on a suitable transportation algorithm can minimize the transportation cost and time. Determining the efficient solution for the large scale of Transportation Problems (TP) is an essential job in the Operation Research (OR). Now-a-days, communication lines, railroad networks, pipeline systems, road networks, shipping lines, aviation lines etc. are typical examples of network. In all these networks, we are interested to send some specific commodity from certain supply places to some demand places. Many of these network flow problems can be formulated as TP. Many researchers have developed a numbers of transportation algorithms and also research works are ongoing for better results. Moreover for finding Initial Basic Feasible Solution (IBFS) much of the research works are concerned with cost matrix and manipulation of cost matrix. It is noted that in TP, all the optimized algorithms initially need an IBFSto obtain the optimal solution.

There are various simple heuristic methods available to get an IBFS, such as, North-West Corner method, Row minimum method, Column minima Method and Least Cost Matrix methodetc.(Taha, 2003). Among all the simple heuristic methods, the Least Cost Matrix (Matrix Minima) is relatively efficient and this method considers the lowest cost cell of the Transportation Table (TT) for making allocation in every stage. There is another well-known algorithm for IBFS is "VAM—Vogel's Approximation Method" (Reinfeld and Vogel,1958). In Vogel's Approximation Method (VAM) penalties are determined from the difference of smallest and next-to-the-smallest cost entries and denoted as Distribution Indicator (DI). Vogel's Approximation Method (VAM) provides comparatively better IBFS. After VAM method, researchers proposed several versions of the VAM method by modifying some tricks such as version of VAM of Shimshak *etal.*(Shimshak *et al.*,1981), Goyal's version of VAM (Goyal, 1984), Ramakrishnan's version of VAM (Ramakrishnan, 1988), Islam's version of VAM(Islam,2012) and Kawser version of VAM (Kawser, 2016) etc.

Kasana *et al.,*2004 introduced the Extreme Difference Method (EDM) in order to find the Initial Basic Feasible Solution (IBFS) of Transportation Problem wherein, the penalties are calculated from the difference of biggest and smallest cost entries. Amirul *et al.*2012 added a new algorithm in calculating the penalties from the difference of highest and near-highest cost elements. Hakim, 2012 introduced an alternative method named as PAM method in calculating the penalties form the difference (Penalty) of maximum and minimum cost elements in each row. Deshmukh, 2012 provided an innovation method named as NMD method in which the algorithm consideredthe subtraction of the minimum odd cost from

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all odd cost to form TOC.Kirca and Satir (1990) developed a heuristic approach, called TOM (Total Opportunity-cost Method), to obtain an initial basic feasible solution for the transportation problem. (Korukoğlu and Balli2011), introduces the Total Opportunity Matrix-TOM (Kasana *et al.*, 2004) which is also new version of VAM. Amirul*et al.*, 2012 proposed a new algorithm by introducing the Total Opportunity Cost Table (TOCT). In order to form the TOCT, he first subtracted the smallest cost entry from each of the entries of every row of the TT and placed them on the right-top of corresponding entry. Then he applied the same operation on each of the column and placed them on the right-bottom of the corresponding entry.

On the other hand very recently, (Azad *et al.*2017), at first developed a TOC (total opportunity cost) table as like Kirca and Satir, 1990 and then they formed DI tableau for allocation by considering the average of TOC of cells along each row identified as Row Average Total Opportunity Cost (RATOC) and the average of TOC of cells along each column identified as Column Average Total Opportunity Cost (CATOC).Allocations of costs are started in the cell along the row or column which has the highest RATOCs or CATOCs.

Recently, Sharma and Bhadane, 2016 presented an alternative method to North West Corner (NWC) method by using Statistical tool called Coefficient of Range (CoR). By considering some numerical examples, they showed that the total transportation cost obtained by CoR method was better than NWC and same as LCM method. The solution by NWC wasDegenerate but by CoR method it was Non-Degenerate.

It is observed that, all the approaches discussed above are concerned with the cost entries and /or the manipulation of cost entries to form DI or TOC table whatever be the structure of supply and demand. None of them considered to treatment in cost elements by manipulating supply/ demand to find DI or TOCin allocation procedures.But it mightbe assumed that supply and demand play a vital role in the formulation of cost allocation table to obtain a better solution. Exploit this idea, in this article; a weighted cost based TOC table is formulated by considering supply and demand entries as a weight factor. Then an algorithm is developed based on Least Cost Matrix method by incorporating weightedopportunity cost. Experiments have been carried out to justify the validity and the effectiveness of the proposed Weighted Opportunity Cost based Least Cost Matrix.

2. FORMULATION OF WEIGHTED COST OPPORTUNITY BASED ALGORITHM

2.1. Mathematical model of balanced transportation problem

Before formulation of weighted opportunity cost matrix based TP algorithm, it is worthwhile to present a mathematical model of a general balanced TP. Let the amount of supply available at source i is a_i and the demand required at destination j is b_j. The cost of transporting one unit from sources i to destination j is c_{ij}. Therefore obviously $a_i \geq 0$ for i and $b_j \geq 0$ for each j. Let x_{ij}be the quantity transported from origin i to destination j. The cost associated with this movement is cost×quantity = $c_{ij}.x_{ij}$. The cost of transporting the commodity from source i, to all destinations, is given by

$$\sum_{j=1}^{n} c_{ij}x_{ij} = c_{i1}x_{i1} + c_{i2}x_{i2} + \cdots + c_{in}x_{in}$$

Thus, the total cost of transporting the commodity from all the sources $(i = 1,2,\cdots,m)$ to all the destinations $(j = 1,2,\cdots,n)$) is

$$\mathbf{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$$

In order to minimize the transportation costs, the general formulation of the transportation problem is as follows:

Minimize $\mathbf{Z} = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}\, x_{ij}$ (Total transportation cost) (2.1)

Subject to $\sum_{j=1}^{n} x_{ij} = a_i\,, i = 1,2,\cdots,m$ (Supplies at origin) (2.2)

$\sum_{i=1}^{n} x_{ij} = b_i \,, i = 1,2,\cdots,n$ (Demands at destination) (2.3)
$x_{ij} \geq 0 \; \forall i, j$ (Quantities) (2.4)

Note that in a balanced transportation model the total supply is equal to the total demand (i.e.$\sum_i^m a_i = \sum_j^n b_j$). The distributions of unit cost as well as demand and supply can be shown in a tabular form given as follows:

Table 2.1: Tabular view of a Transportation Problem (TP)

	Destinations						
		D_1	D_2	$\cdots$	$\cdots$	D_n	Supply
Origins	O_1	c_{11}	c_{12}	$\cdots$	$\cdots$	c_{1n}	a_1
	O_2	c_{21}	c_{22}	$\cdots$	$\cdots$	c_{2n}	a_2
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\cdots$		
	$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\cdots$		
	O_m	c_{m1}	c_{m2}	$\cdots$	$\cdots$	c_{mn}	a_m
	Demand	b_1	b_2	$\cdots$	$\cdots$	b_n	
	Requirement						

2.2 Formulation of Weighted Opportunity Cost (WOC) Table and Algorithm

It is a role of thumb that maximum supply will be done where transportation cost is minimum. Upon this idea researcher developed the well-known Least Cost Matrix method to find optimal solution in TP. But reality is that, maximum time it does not provide optimal solution or near optimal solution at all. Moreover, most of the time it provides the IBFS which is not good enough too. It is also a general practice in business arena that among the several demands shopkeeper want to sell where demand is maximum so that he can able to sell maximum with saving several sell parameters such as time. Therefore amount of supply and demand could play a vital role in business arena including TP. Exploit theseconcepts; we will first develop a Weighted Opportunity Cost (WOC) table. Then on the base of Least Cost Matrix method we will develop a new approach to solve TP by incorporating the WOC table.

At first, will try to formulate a Weighted Opportunity Cost (WOC) table in which supply and demand will be treated as weight factors on cell cost (transportation cost) of the cost matrix. For the formulation of the WOC as well as algorithm for solving TP we have encountered the following steps:

Step1 (finding cell weight): At first, we have find out the maximum possible allocation of the cell C_{ij}, which is indeed *min* (S_i, D_j), where S_i denotes total supply at node *i* and $D_j$indicates total demand at node *j*. Therefore total possible allocation will be as follow: $\sum_{i=1}^{n}\sum_{j=1}^{n} min(S_i, D_j)$

Therefore for each C_{ij},,its cell's weight will be $min\,(S_i, D_j)/\sum_{i=1}^{n}\sum_{j=1}^{n} min(S_i, D_j)$.

Step 2 (Apply weight to each cell): Now as in least cost matrix method as well as in natural role of sense, smaller cost cell has larger priority for allocation, so we have form a virtual weighted cost at cell C_{ij}, as $min\,(S_i, D_j)/\sum_{i=1}^{n}\sum_{j=1}^{n} min(S_i, D_j) \times \frac{1}{c_{ij}}$ such that each cell cost $c_{ij} \neq 0$. In the case of $c_{ij} = 0$, we put a very large value $M \times min\,(S_i, D_j)$ such that $M > max\left\{\frac{S_i,D_j}{\sum_{i=1}^{n}\sum_{j=1}^{n} min(S_i,D_j)} \times \frac{1}{c_{ij}}; \forall i, j \text{ and } c_{ij} \neq 0\right\}$. Note that for more than one zero cell's cost, the weighted cost will be different according to the value of cell's row supply and column demand i.e. $min\,(S_i, D_j)$. So if there are more than one $c_{ij} = 0$, among the all zero cell cost, we have obviously obtained larger weighted cost in that cell where $min\,(S_i, D_j)$ is larger.

Step 3 (Allocation procedure): Allocation procedure is very simple, similar to the Least Cost Matrix approach but here we consider weighted cost rather than exact cost. That is, allocate to the cell corresponding to ***maximum weighted cost*** rather than ***minimum cost***. But if more than one cell weighted cost are identical then allocate in which cell cost **is minimum**. Again if more than one cell weighted cost are identical and also cell cost are identical (then off course for all cases the $min\{D_i, S_j\}$ values are identical) then tie one of them arbitrarily. It is noted that after allocation to the cell C_{ij} contained the maximum weighted cost the cell C_{ij} will be ignore along with weighted cost. Moreover all the cells of the row *i* if $S_j = min\{D_i, S_j\}$ or column *j* if $D_i = min\{D_i, S_j\}$ or both row *i* and column *j* if $S_j = D_i = min\{D_i, S_j\}$ are ignored after allocation at the cell C_{ij} with amount $min\{D_i, S_j\}$. So for further allocation if any we need to consider reduced cell.

It is worthwhile to mention here that the weighted cost $min\ (S_i, D_j)/\sum_{i=1}^{m}\sum_{j=1}^{n} min(S_i, D_j) \times \frac{1}{c_{ij}}$ or $min\ (S_i, D_j) \times \frac{1}{c_{ij}}$ provides identical information regarding comparison among cell weighted cost. So we have form a virtual weighted cost at cell $C_{ij,}$ as $min\ (S_i, D_j \times \frac{1}{c_{ij}}$ such that each cell cost $c_{ij} \neq 0$ rather than $(S_i, D_j)/\sum_{i=1}^{m}\sum_{j=1}^{n} min(S_i, D_j) \times \frac{1}{c_{ij}}$. So it is reduced a significant computational cost.

Step 4 (Termination): Continuing the allocation procedure of Step 3 sequentially until all possible allocations are done.

3. EXPERIMENTAL RESULTS AND DISCUSSIONS

3.1 Experiment with elaborated discussion:

For the illustration of the proposed weighted opportunity cost table as well as proposed weighted Least Cost Matrix method, we first consider a typical problem 1 given below:

Example 1:

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	1	2	3	4	5	20
O_2	2	5	9	4	3	20
O_3	3	6	5	7	6	15
O_4	4	2	4	6	7	15
Demand	5	6	14	20	25	

Solution: Note that the given problem is balanced TP problem, since total demand =total supply= 70. At firstwe need to form weighted opportunity cost (WOC) table. Since there is no any zero cost cellsso we need not consider M. Therefore the WOC table of the problem 1 is shown in Table 3.1.

Step 1(Formulation WOC table):

For simplicity of visualization we have incorporated the WOC table in given TT and displayed in Table 3.2. It is observed that in each cell top left indicates weighted cost whereas top right indicates actual cost. Now we have to allocate in the cell which contains **largest weighted cost**.

Table 3.1: Weighted Opportunity Cost (WOC) table of the problem 1

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	5/1	6/2	14/3	20/4	20/5	20
O_2	5/2	6/5	14/9	20/4	20/3	20
O_3	5/3	6/6	14/5	15/7	15/6	15
O_4	5/4	6/2	14/4	15/6	15/7	15
Demand	5	6	14	20	25	

Step 2 (1st allocation): It is observed in the Table 3.2 that the maximum weighted cell cost is **20/3** contained in the cell C_{25}. Therefore we have to allocate amount of min {20, 25} i.e. 20 at the cell C_{25} and we have re-adjusted available supply of row 2 which is vanished as well as column 5 which is 25-20=5. Moreover we have to ignore all the cell of rows 2 as there is no any supply item. The pictorial view after 1st allocation shown in the Table 3.3.

Table 3.2: WOC based transportation table for the problem 1

	D_1		D_2		D_3		D_4		D_5		Supply
O_1	5/1	1	6/2	2	14/3	3	20/4	4	20/5	5	20
O_2	5/2	2	6/5	5	14/9	9	20/4	4	**20/3**	3	20
O_3	5/3	3	6/6	6	14/5	5	15/7	7	15/6	6	15
O_4	5/4	4	6/2	2	14/4	4	15/6	6	15/7	7	15
Demand	5		6		14		20		25		

Table 3.3: WOC based transportation table for the problem 1: after 1[st] allocation

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	**5.0** 1	3.0 2	14/3 3	5.0 4	4.0 5	20
O_2	5/2 2 ×	6/5 5 ×	14/9 9 ×	5.0 4 ×	20/3 3 **20**	~~20~~
O_3	5/3 3	6/6 6	14/5 5	15/7 7	15/6 6	15
O_4	5/4 4	6/2 2	14/4 4	15/6 6	15/7 7	15
Demand	5	6	14	20	~~25~~, 5	

Table 3.4: WOC based transportation table for the problem 1: after 2[nd] allocation

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	5.01 **5**	3.0 2	14/3 3	**5.0** 4	4.0 5	~~20~~, 15
O_2	5/2 2 ×	6/5 5 ×	14/9 9 ×	5.0 4 ×	20/3 3 **20**	~~20~~
O_3	5/3 3 ×	6/6 6	14/5 5	15/7 7	15/6 6	15
O_4	5/4 4 ×	6/2 2	14/4 4	15/6 6	15/7 7	15
Demand	~~5~~	6	14	20	~~25~~, 5	

Step 3 (2nd allocation): After 1[st] allocation, the reduced space for allocation is shown in the Table 3.3,it is observed in the table that the remaining maximum weighted cell cost is 5.0 contained in the two cells namely C_{11} and C_{14} (here the cell C_{24} is ignored) but whose actual cost are 1 and 4 respectively. Therefore we have to allocate amount of min {5, 20} i.e. 5 at the cell C_{11} (as its actual cost is minimum) and we re-adjust available supply of row 1 which is 15 as well as column 1 which is vanished. The tabular view of reduced weighted opportunity cost table is displayed at Table 3.4.

Step 4 (3rd allocation): It is observed that the maximum weighted cell cost is 5 contained in the cell C_{14} in the reduced WOC matrix given in the Table 3.4. So we have allocated the amount of min {20,15} =15 at the cell C_{14}. Then update the reduced matrix which is shown at Table 3.5.

Table 3.5: WOC based transportation table for the problem 1: after 3[rd] allocation

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	5.01 **5**	3.0 2 ×	14/3 3 ×	5.0 4 **15**	4.0 5 ×	~~20~~, ~~15~~
O_2	5/2 2 ×	6/5 5 ×	14/9 9 ×	5.0 4 ×	20/3 3 **20**	~~20~~
O_3	5/3 3 ×	6/6 6	14/5 5	15/7 7	15/6 6	15
O_4	5/4 4 ×	6/2 2	**14/4** 4	15/6 6	15/7 7	15
Demand	~~5~~	6	14	~~20~~, 5	~~25~~, 5	

Table 3.6: WOC based transportation table for the problem 1: after 4[th] allocation

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	5.01 **5**	3.0 2 ×	14/3 3 ×	5.0 4 **15**	4.0 5 ×	~~20~~, ~~15~~
O_2	5/2 2 ×	6/5 5 ×	14/9 9 ×	5.0 4 ×	20/3 3 **20**	~~20~~
O_3	5/3 3 ×	6/6 6	14/5 5 ×	15/7 7	15/6 6	15
O_4	5/4 4 ×	**6/2** 2	14/4 4 **14**	15/6 6 ×	15/7 7 ×	~~15~~, 1
Demand	~~5~~	6, 5	~~14~~	~~20~~, 5	~~25~~, 5	

Step 5 (4th allocation): It is observed that the maximum weighted cost is now **14/4**at the cell C_{43} in the reduced WOC matrix given in the Table 3.5. So we have allocated the amount of min {14,15} =14 at the cell C_{43} and then update the reduced matrix which is shown at Table 3.6.

Step 6 (5th allocation): It is observed that the maximum weighted cost is now **6/2** at the cell C_{42} in the reduced WOC matrix given in the Table 3.6. So we have allocated the amount of min {6,1} =1 at the cell C_{42} and then update the reduced matrix which is shown at Table 3.7.

Step 7 (Allocations to remain cells): It is now observed that after 5th allocation the reduced WOC matrix has no alternative cell to choose (see Table 3.7). Therefore we have to allocated amount 5, 5 and 5 to the cellsC_{32}, C_{34} and C_{35} respectively. Therefore after allocations to all possible cells we have allocated complete transportation table as Table 3.8.

Table 3.7: WOC based transportation table for the problem 1: after 5th allocation

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	5.01 **5**	3.0 2 ×	14/3 3 ×	5.0 4 **15**	4.0 5 ×	~~20~~, ~~15~~
O_2	5/2 2 ×	6/5 5 ×	14/9 9 ×	5.0 4 ×	20/3 3 **20**	~~20~~
O_3	5/3 3 ×	6/6 6	14/5 5 ×	15/7 7	15/6 6	15
O_4	5/4 4 ×	6/2 2 **1**	14/4 4 **14**	15/6 6 ×	15/7 7 ×	~~15~~, ~~1~~
Demand	~~5~~	~~6~~, 5	~~14~~	~~20~~, 5	~~25~~, 5	

Table 3.8: WOC based transportation table for the problem 1: after all allocations

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	5.01 **5**	3.0 2 ×	14/3 3 ×	5.0 4 **15**	4.0 5 ×	~~20~~, ~~15~~
O_2	5/2 2 ×	6/5 5 ×	14/9 9 ×	5.0 4 ×	20/3 3 **20**	~~20~~
O_3	5/3 3 ×	6/6 6 **5**	14/5 5 ×	15/7 7 **5**	15/6 6 **5**	~~15~~
O_4	5/4 4 ×	6/2 2 **1**	14/4 4 **14**	15/6 6 ×	15/7 7 ×	~~15~~, ~~1~~
Demand	~~5~~	~~6~~, ~~5~~	~~14~~	~~20~~, ~~5~~	~~25~~, ~~5~~	

Therefore according to **the proposed WOC based Least Cost Matrix** method:

Total Transportation Cost =5×1 +15×4+20×3+5×6+5×7+5×6+1×2+14×4= 278.

Also we have corresponding 8 basic variables: $\{x_{11}, x_{14}, x_{25}, x_{32}, x_{34}, x_{35}, x_{42}, x_{43}\}$which are all non-degenerated (since m+n-1=. 4+5-1=8).

Now we have solved the Problem 1 by using Least Cost Matrix method. After allocations to all possible cells by Least Cost Matrix method we have the allocated table: Table 3.9.Therefore according to the **Least Cost Matrix** method:

Total Transportation Cost =5×1 +6×2+9×3+20×3+10×7+5×6+5×4+10×6= 284.

Also we have again corresponding 8 basic variables: $\{x_{11}, x_{12}, x_{13}, x_{25}, x_{34}, x_{35}, x_{43}, x_{44}\}$ which are all non-degenerated (since m+n-1=. 4+5-1=8).

It is noticed that the proposed WOC based **Least Cost Matrix** method performs better to Least Cost Matrix method. It is also observed that the some of the basics variables of the two approaches are different.

Table 3.9: The Least Cost Matrix method of the problem 1: After all allocations

	D_1	D_2	D_3	D_4	D_5	Supply
O_1	1 **5**	2 **6**	3 **9**	4 ×	5 ×	~~20~~,~~15~~,~~9~~
O_2	2 ×	5 ×	9 ×	4 ×	3 **20**	~~20~~
O_3	3 ×	6 ×	5 ×	7 **10**	6 **5**	~~15~~, ~~5~~
O_4	4 ×	2 ×	4 **5**	6 **10**	7 ×	~~15~~, ~~10~~
Demand	~~5~~	~~6~~	~~14~~, ~~5~~	~~20~~, ~~10~~	~~25~~, ~~5~~	

3.2 Further experiments and discussions:

Now for further analysis of the performance of proposed method, we consider some more instances given in the first column of the Table 3.10. After carry on the experiments upon the instances (given in the Table 3.10) by using proposed method as well as Least Cost Matrix method, the experimental results are incorporated in the Table 3.10. In the Table 3.10 LCM is stood for Least Cost Matrix method and WOC_LCM denotes Weighted Opportunity Cost based LCM approach. It is observed in the table that out of 7 instances, in 4 instances (Ex. No. 1, 2, 5 and 6), the proposedWOC based Least Cost matrix method outperforms compared to Least Cost matrix method. In two cases (Ex. No. 3 and 4) both results are identical.

Table 3.10: Comparison of WOC_LCM and LCM approaches Transportation Problems

Example No.	**Problem**	**WOC_LCM**	**LCM**
2	c_{ij}:{(1,2,3,1,2,3,1,2,3); (0,3,2,0,1,2,1,2,1); (2,1,0, 1,2,1,2,1,1); (1,1,1,2,2,2,2,2,1)} S: (15,15,12,8) D: (2,8,4,15,11,2,3,4,1)	**43**	44
3	c_{ij}:{(1,2,3,1,2,3,1,2,3); (0,3,2,0,1,2,1,2,1); (2,1,0, 1,2,1,2,1,1); (1,1,1,2,2,2,2,2,1)} S: {15,5,12,18} D: {12,8,14,5,1,2,3,4,1}	**48**	50
4	c_{ij}:{(2,4,1,3); (4,3,5,2); (5,2,3,6)} S: {10,20,10} D:{9,11,6,14}	85	85
5	c_{ij}:{(1,2,3,1,2,3,1,2,3); (0,3,2,0,1,2,1,2,1); (2,1,0, 1,2,1,2,1,1); (1,1,1,2,2,2,2,2,1)} S: {10,20,12,8} D: {2,3,4,5,1,2,3,14,16}	48	48
6	c_{ij}:{(2,4,3); (12,5,8); (4,2,3)} S: {5,20,16} D: {4,20,7}	**129**	141
7	c_{ij}:{(2,1,4); (1,2,5); (5,15,3)} S: {22,9,7} D: {23,5,10}	**136**	156

It is remarks that the instances in which proposed method performed better, because of weighted opportunity cost play a significance role. On the other hand in the case of identical results, it is notice that weighted factors have no impact on allocation flow. That is weighted opportunity cost table and without weighted opportunity cost table is identical regarding opportunity of allocations. Few experiments have also been carried out for the comparison of VAM method. By comparing the experimental results, it may say that, the proposed method is also comparable with VAM method in the cases when WOC table has significant impact.

4. CONCLUSION

In this article, we have proposed a Weighted Opportunity Cost based Least Cost Matrix approach (WOC-LCM) by incorporating Weighted Opportunity Cost in Least Cost Matrix method. Actually it is a modified

Least Cost Matrix method in which flow of allocation is done by Weighted Opportunity Cost matrix. The significant contribution of this research work is the formulation of Weighted Opportunity Cost matrix by considering demand and supply as a weight factor which is off course a unique idea. The performance of the proposed method is also significantly good enough compared to Least Cost Matrix method. Moreover, it is also hoped that, in future, researchers will able to obtain some nice approaches to solve TP by exploiting the concept of the proposed WOC.

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