Journal of Engineering Science 08(1), 2017, 41-50

JES
an international Journal

EFFECT OF INTERPARTICLE FRICTION ANGLE ON THE STRESS-DILATANCY RESPONSES OF GRANULAR ASSEMBLY BY DEM

Md. Mahmud Sazzad[1*], Golam Asibul Alam[2] and Md. Nurul Alam Rowshan[3]

[1]*Department of Civil Engineering, Rajshahi University of Engineering & Technology, Bangladesh*
[2]*Bangladesh Accord, Bangladesh*
[3]*Bangladesh University of Engineering & Technology, Bangladesh*
[*]*E-mail address:*
Received: 14 October 2015 Accepted: 18 June 2017

ABSTRACT

Numerical study was performed to investigate the stress-dilatancy behavior of granular materials such as sand due to the variation of interparticle friction angle (ϕ_μ) using the discrete element method (DEM). Spheres were randomly generated in a cube and compressed isotropically. Samples of different densities were prepared virtually in the laboratory for different confining pressures. Simulations of true triaxial compression tests were carried out using the isotropically compressed dense and loose samples under strain controlled conditions for different ϕ_μ and confining pressures. The simulated stress-dilatancy responses depict good qualitative agreement with the experimental results at the macroscopic scale. However, stress ratio does not approach to a unique state (i.e. critical state) even at a very large strain when ϕ_μ is smaller (0.1° - 10°) regardless of the confining pressures and sample densities. Dense sample behaves like a loose sample when ϕ_μ is close to zero regardless of the confining pressure.

Keywords: *Discrete Element Method, Interparticle Friction Angle, Confining Pressure, Dilatancy*.

1. INTRODUCTION

The stability of a granular system extensively depends on ϕ_μ. It is one of the prominent properties of granular materials that have a crucial role in resisting shear. Consequently, ϕ_μ influences the mechanical responses of a granular system. Experimentally, Skinner (1969) was probably the first to investigate the behavior of a granular system for different ϕ_μ by shearing a random assembly of spherical particles. The study indicated that the effective angle of shearing resistance at peak and residual state do not increase monotonically with the increase of ϕ_μ for a given initial porosity. Triaxial tests on an aggregate of steel spheres were also reported in Suiker and Fleck (2004). Their results at lower values of ϕ_μ differ from that of Skinner (1969). Using DEM, a number of studies have been carried out to examine the influence of ϕ_μ both on the macro and micro-mechanical responses (Thornton 2000, Rothenburg and Kruyt 2004, Hu and Molinari 2004, Powrie *et al.* 2005, Kruyt and Rothenburg 2006, Sazzad and Islam 2008, Antony and Kruyt 2009, Sazzad and Suzuki 2010, Barreto and O'Sullivan 2012). Of these studies, Sazzad and Islam (2008) reported the influence of ϕ_μ on the formation of shear band in a granular system. Their results indicated that the formation of shear band is a function of ϕ_μ and a clear cross band is noticed for larger values of ϕ_μ. Sazzad and Suzuki (2010) depicted that ϕ_μ significantly affects the macro-micro characteristics of granular materials during cyclic loading. Recently, Barreto and O'Sullivan (2012) reported the influence of ϕ_μ on soil response under the generalized stress condition. Even though, many numerical studies have been performed, the influence of ϕ_μ on stress-dilatancy behavior at lower confining pressures in a loose sample is not known. It is worthy of noting that, investigating the influence of ϕ_μ through experiment is quiet difficult, particularly at lower confining pressures for loose samples. Consequently, the objective of the present study is to examine the influence of ϕ_μ on the stress-dilatancy behavior of a granular system by DEM at lower confining pressures considering both the dense and loose samples. Numerical samples were randomly generated in a cube without any overlap. These initially generated samples were compressed

* Corresponding Author: mmsruet@gmail.com

isotropically until the desired density and confinement were attainted. A series of true triaxial compression tests were conducted on cubic samples consisting of spheres under strain controlled condition with a very small vertical strain increment using DEM. Digital data were recorded at regular intervals during the simulation of true triaxial compression tests for the post-process. The digital data were analyzed and the stress-dilatancy responses of granular materials for different ϕ_μ under varying confining pressures and sample densities have been reported.

2. DEM AND YADE

DEM was introduced by Cundall and Strack (1979) for granular materials. In DEM, each particle is considered as an element and liable to sliding and rotation upon the applied load. Each particle is able to make or break contacts with its neighbors. In this process, Newton's second law is used to calculate the accelerations of particles. Translational and rotational accelerations are integrated twice over time to get the displacement of particle. Force displacement law is also used to get the force from displacement and the cycle continues. The transitional and rotational accelerations were computed using the following equations as follows:

$$m\ddot{x}_i = \sum F_i \quad i = 1-3 \tag{1}$$

$$I\ddot{\theta} = \sum M \tag{2}$$

where m is the mass, $\ddot{x}_i$ are the translational accelerations, F_i are the force components, I is the moment of inertia, $\ddot{\theta}$ is the rotational acceleration and M is the moment.

The simulation was carried out using the computer code YADE (Koziki and Donze 2008) based on DEM. It was written by C++ language. New numerical models can be added in the code by plugging in the related formulas. For details of YADE, readers are referred to Koziki and Donze (2008). A linear spring-dashpot model is used in YADE. The resultant normal and shear stiffness's, k_n and k_s, respectively, of two contacting particles (spheres) A and B are given by

$$k_n = \frac{2Er^A r^B}{r^A + r^B} \tag{3}$$

$$k_s = \alpha k_n \tag{4}$$

where E is the contact Young's modulus, r^A and r^B are the radii of particles A and B, respectively, α is the stiffness ratio (k_s / k_n).

3. SAMPLE PREPARATION METHOD

In the present study, three different confining pressures were used, each of which considered both the dense and loose samples. Under dense or loose sample, five different ϕ_μ were used. To prepare dense samples for a particular confining pressure, around 4000 particles were randomly generated in a cubic box setting ϕ_μ to zero. The assembly was then subjected to isotropic compression under the desired confining pressure until stress oscillation becomes negligible. Later, five dense assemblies were prepared from this sample setting ϕ_μ to 0.1°, 10°, 20°, 30° and 40°, respectively until the porosity of the assemblies became constant for the last few thousand steps. Similar procedure was adopted to prepare loose assembly except that ϕ_μ was set to 26.5° instead of zero during the random generation of particles and first phase of isotropic compression. It should be noted that the lower values of ϕ_μ are considered in this study to examine the effect of ϕ_μ on the stress-dilatancy behavior of granular materials even though such lower values of ϕ_μ are practically insignificant. The isotropically compressed dense sample for a given confining pressure is depicted in Figure 1 with reference axes.

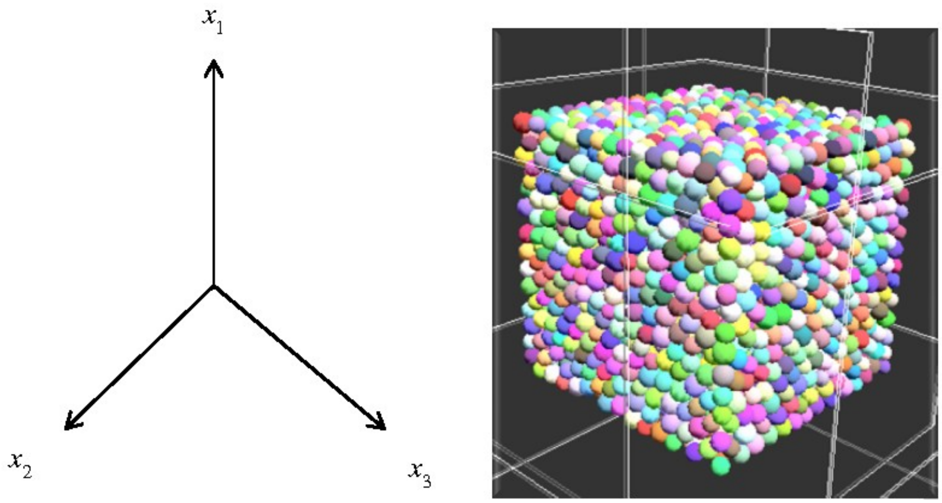

Figure 1: The isotropically compressed dense sample for a given confining pressure with reference axes

4. NUMERICAL SIMULATIONS

Simulations of true triaxial compression test were conducted under strain controlled condition by moving the top and bottom boundaries inward the sample with a small strain rate while keeping the lateral stresses in other directions constant by continuously adjusting the position of other four boundaries of the sample. The simulation parameters used in the study are given in Table 1.

Table 1: Parameters used in the simulations

Simulation parameters	**value**
Young Modulus (N/m^2)	60×10^6
Mass density (kg/m^3)	2600
Stiffness ratio, α	0.50
Strain rate	0.1

5. STRESS-STRAIN RESPONSES

The stimulated stress-strain behavior for true triaxial compression test under different ϕ_μ for dense and loose samples having a confining pressure of 25 kPa is depicted in Figure 2. Stress ratio, σ_1/σ_3, [σ_1 and σ_3 are the stresses in $x_1-$and x_3-direction, respectively] gradually increases with axial strain, ε_1, for lower values of ϕ_μ; however, σ_1/σ_3 attains a peak followed by huge strain softening for higher values of ϕ_μ in a dense sample. This happens because the dense sample becomes stronger with the increase of ϕ_μ. Here, it can be noted that the peak value of σ_1/σ_3 increases continually with the increase of ϕ_μ. In case of loose sample, no apparent strain softening is noticed even for very higher values of ϕ_μ (e.g. ϕ_μ=40˚). One interesting point is that σ_1/σ_3 does not depict uniqueness (i.e. approach to a critical state) even at ε_1=25% (residual state) when ϕ_μ is in smaller ranges (e.g., ϕ_μ=0.1˚ or 10˚). This phenomenon reveals that a granular sample does not reach a critical state when ϕ_μ is smaller regardless of sample density. The continuous collapse of force chains and consequently, the continuously instability of the system for lower values of ϕ_μ is linked to this phenomenon. Similar behavior is noticed in Figures 3 and 4, respectively, for other values of confining pressures (i.e. 50 and 100 kPa) except the elevated values of

σ_1/σ_3 compared to the confining pressure of 25 kPa. This reveals that the overall behavior is not a function of confining pressures.

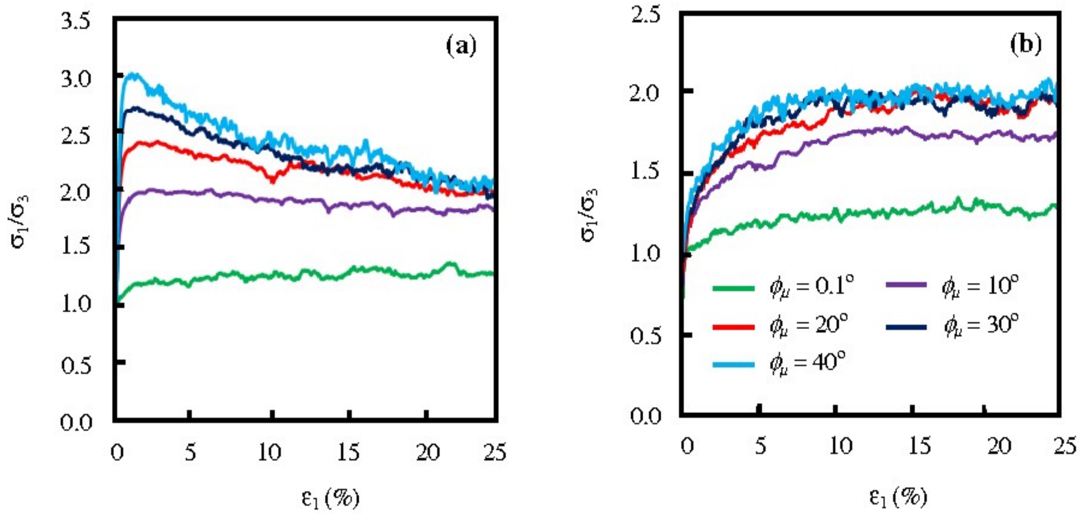

Figure 2: Effect of ϕ_μ on stress-strain responses for a confining pressure of 25 kPa: (a) dense sample, (b) loose sample

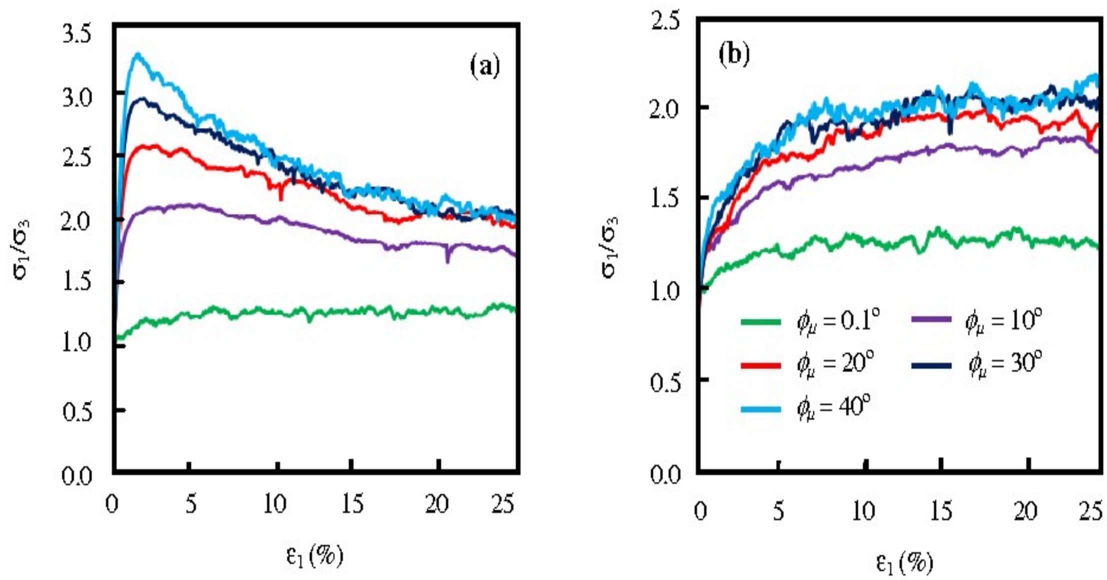

Figure 3: Effect of ϕ_μ on the stress-strain responses for a confining pressure of 50 kPa: (a) dense ample, (b) loose sample

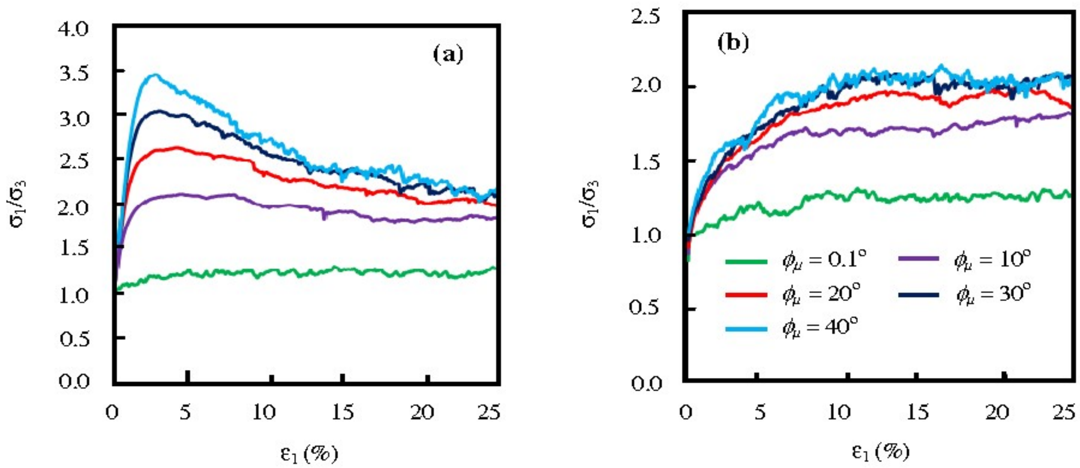

Figure 4: Effect of ϕ_μ on the stress-strain responses for a confining pressure of 100 kPa: (a) dense sample, (b) loose sample

A comparison between the angle of internal friction, $\phi = \sin^{-1}[(\sigma_1 - \sigma_3)/(\sigma_1 + \sigma_3)]$, observed in this study at ε_1=25% for dense sample and the experimental result by Suiker and Fleck (2004) is depicted in Figure 5. It can be noted that the simulated results have nice qualitative agreement with the experimental result. This obviously indicates the versatility of the present simulation.

A comparison between the simulated results and the theoretical results by Bishop (1954) is depicted in Figure 6. Bishop (1954) derived an approximate solution for triaxial compression test as $\phi = \sin^{-1}[15\mu/(10+3\mu)]$, where $\mu = \tan\phi_\mu$. It can be noted that disagreement is noted for all values of ϕ_μ. This disagreement between the simulated and the theoretical results is associated with the exclusion of particle rolling as a permissible mechanism in the theoretical derivation by Bishop (1954).

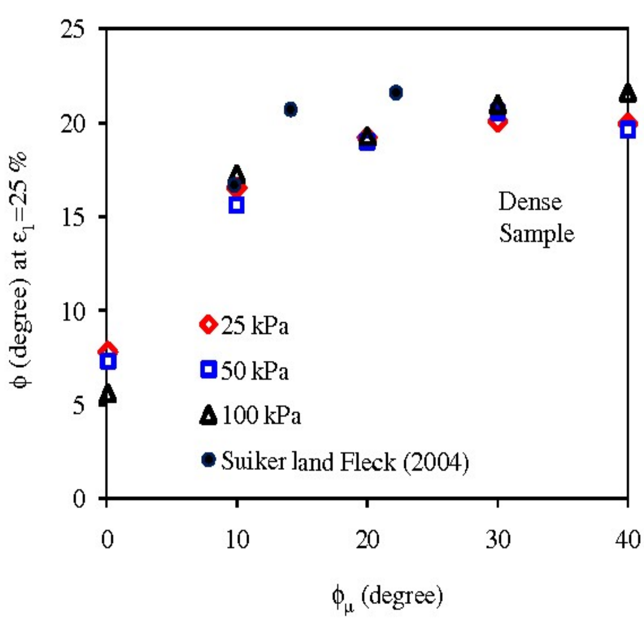

Figure 5: A comparison between the simulated results and the experimental results by Suiker and Fleck (2004)

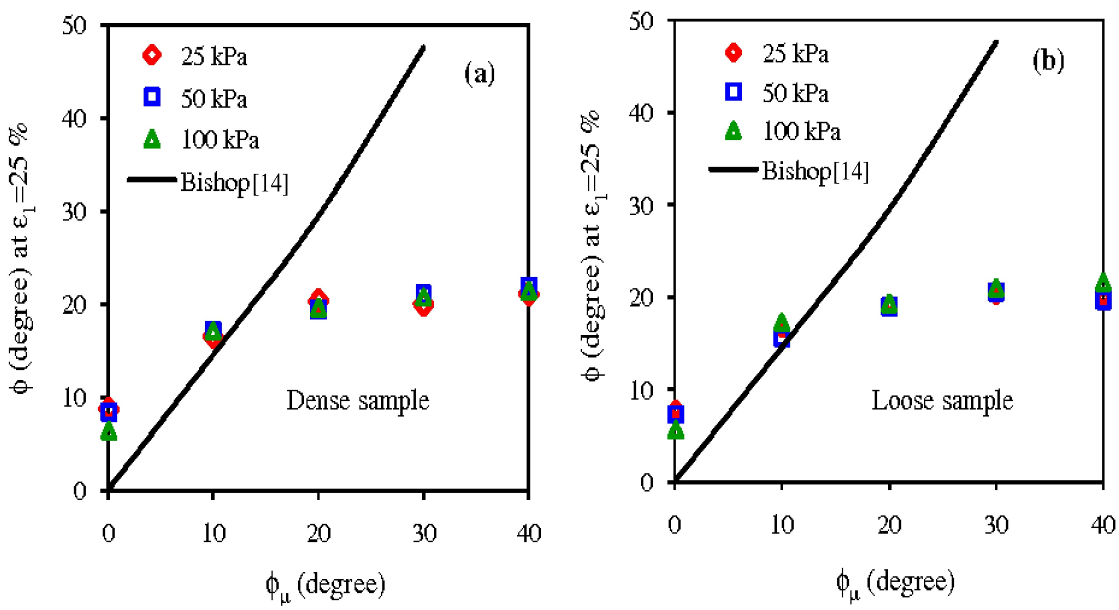

Figure 6: A comparison between the simulated results and the theoretical results by Bishop (1954): (a) dense sample, (b) loose sample

6. DILATIVE RESPONSES

The evolution of volumetric strain against the axial strain for a confining pressure of 25 kPa in dense and loose samples is depicted in Figure 7. The volumetric strain is defined as $\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$, where ε_1, ε_2 and ε_3 is the strains in x_1-, x_2- and x_3- direction, respectively. A positive value of ε_v indicates compression while a negative value indicates dilation. No significant change in ε_v is noticed for a dense sample under a confining pressure of 25 kPa when ϕ_μ is 0.1°. The behavior is rather compressive. However, huge dilation is observed with axial strain for other values of ϕ_μ. Here, it can also be noted that ε_v increases with the increase of ϕ_μ. For loose sample, on the other hand, compressive behavior is observed as expected. Maximum compression is noticed for ϕ_μ is 0.1° and it reduces as ϕ_μ increases. Similar behavior is observed in Figures 8 and 9 for other values of confining pressures of 50 and 100 kPa, respectively. It suggests that the evolution tendency of ε_v for the variation of ϕ_μ is not pressure dependent.

The evolution of dilatancy index, defined as $DI = -d\varepsilon_v / d\varepsilon_1$, is depicted in Figure 10. Here, $d\varepsilon_v$ is the change in ε_v and $d\varepsilon_1$ is the change in ε_1. The behavior is depicted only for a confining pressure of 25 kPa because the similarity is noticed for other confining pressures. It can be noted that, the shape of the dilatancy index versus axial strain curve is similar to that of stress ratio versus axial strain curve for dense sample. Dilatancy index increases with the increase of ε_1 up to peak and then approaches to almost zero at larger strain for higher values of ϕ_μ. In loose sample, dilatancy index gradually increases until it reaches almost a zero value.

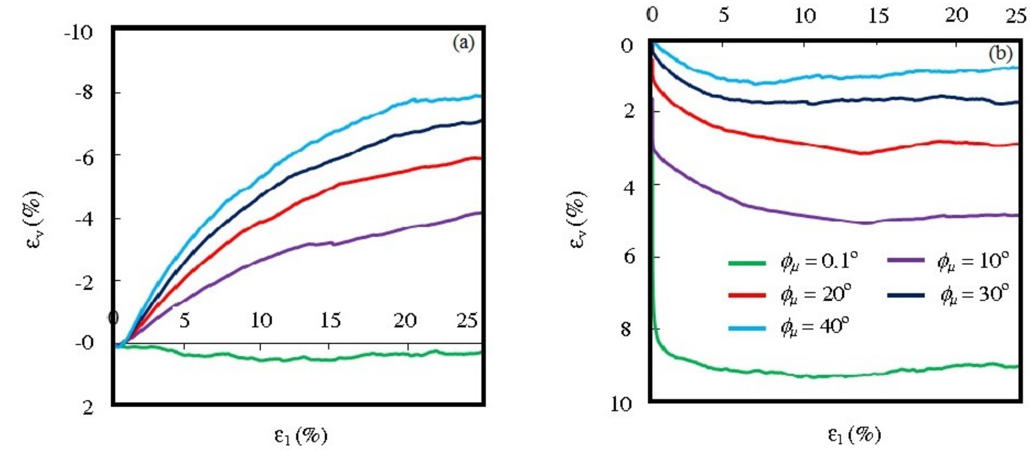

Figure 7: Effect of ϕ_μ on the volumetric strain for 25 kPa: (a) dense sample, (b) loose sample

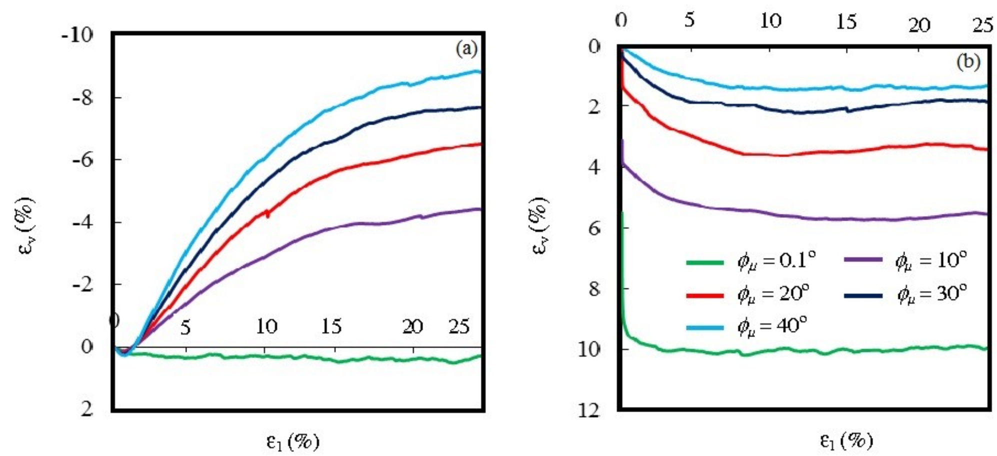

Figure 8: Effect of ϕ_μ on the volumetric strain for 50 kPa: (a) dense sample, (b) loose sample

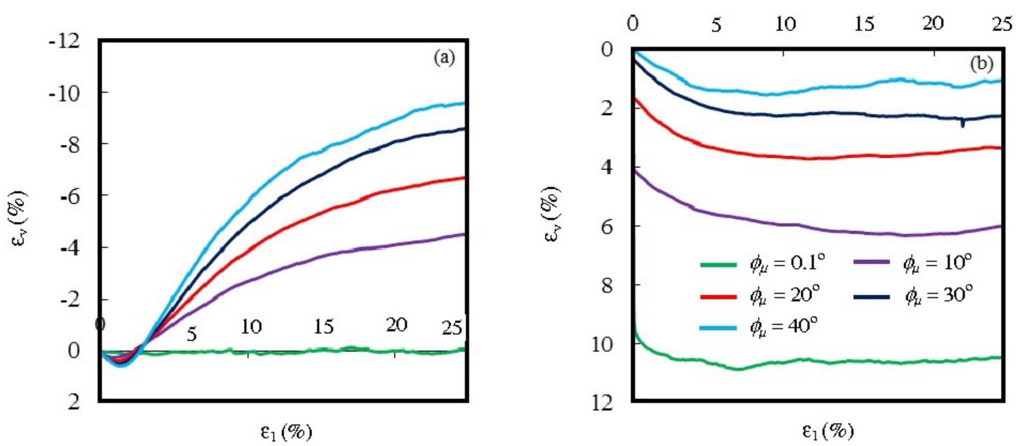

Figure 9: Effect of ϕ_μ on volumetric strain for 100 kPa: (a) dense sample, (b) loose sample

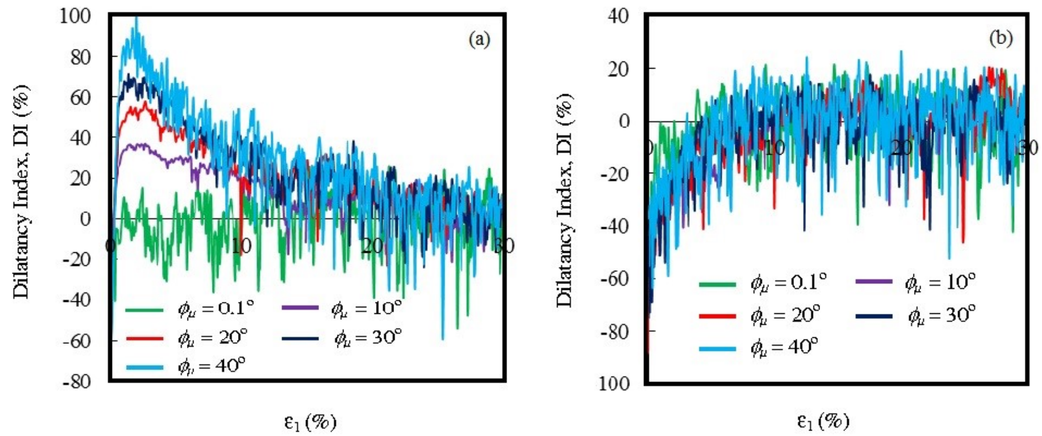

Figure 10: Evolution of dilatancy index for different values of ϕ_μ at a confining pressure of 25 kPa: (a) dense sample, (b) loose sample

The relationship between $-d\varepsilon_v / d\varepsilon_1$ and $(\sigma_1/\sigma_3)_{peak} - (\sigma_1/\sigma_3)_{at\ 25\%\ strain}$ for dense sample considering the confining pressures of 25, 50 and 100 kPa is depicted in Figure 11. It can be noted that a linear relationship exists between $-d\varepsilon_v / d\varepsilon_1$ and $(\sigma_1/\sigma_3)_{peak} - (\sigma_1/\sigma_3)_{at\ 25\%\ strain}$ for dense sample. The corelation between $-d\varepsilon_v / d\varepsilon_1$ and $(\sigma_1/\sigma_3)_{peak} - (\sigma_1/\sigma_3)_{at\ 25\%\ strain}$ can be expressed as follows:

$$\frac{-d\varepsilon_v}{d\varepsilon_1} = 0.702\left[(\sigma_1/\sigma_3)_{peak} - (\sigma_1/\sigma_3)_{at\ 25\%\ strain}\right] + 0.223 \quad (5)$$

It can be noted that equation (5) is not a function of ϕ_μ. It suggests that the origin of equation (5) may be geometric in nature as stated in Kruyt and Rothenburg (2006).

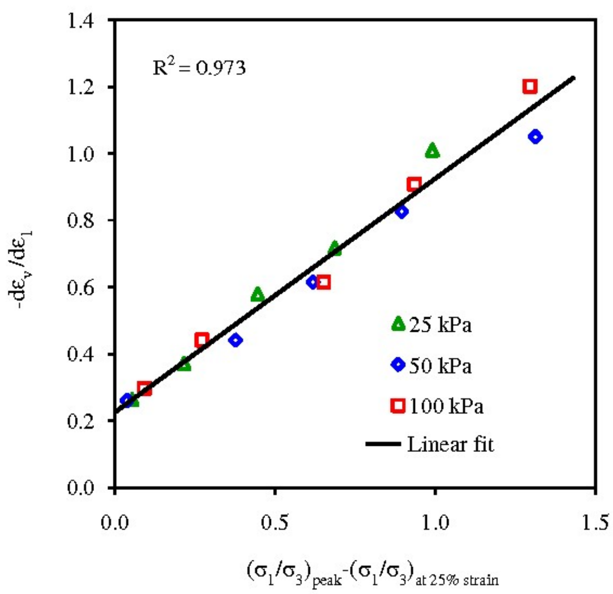

Figure 11: Relationship between $-d\varepsilon_v / d\varepsilon_1$ and $(\sigma_1/\sigma_3)_{peak} - (\sigma_1/\sigma_3)_{at\ 25\%\ strain}$ for dense sample

7. CONCLUSIONS

In this study, numerical simulations were carried out to investigate the influence of ϕ_μ on the stress-dilatancy characteristics of sand as granular materials at low confining pressures particularly in loose assemblies. Several important points of the numerical study are summarized as follows:

i) The stress ratio attains a peak followed by huge strain softening for higher values of ϕ_μ in a dense sample while no apparent strain softening is noticed even for very higher values of ϕ_μ (e.g. ϕ_μ=40° in case of loose sample). These simulated macro results are qualitatively consistent to that usually observed in experimental results.

ii) In case of loose sample, no strain softening is noticed even for very higher values of ϕ_μ (e.g. ϕ_μ=40°). This behavior is not a function of confining pressures.

iii) Stress ratio does not approach to a unique state even at very large strain when ϕ_μ is smaller (0.1° - 10°) regardless of the confining pressures and sample densities.

iv) Dense sample behaves like a loose sample when ϕ_μ is close to zero regardless of the confining pressure. This phenomenon is obviously due to almost zero values of ϕ_μ between contacting particles that restrict them to build stable force chains.

v) The shape of the dilatancy index-axial strain curves has similarity with the stress-ratio-axial strain curves.

vi) A linear correlation between dilatancy index and the differences of stresses at peak and residual state for dense sample regardless of confining pressure is established. ϕ_μ. It is suggested that the origin of linear correlation between dilatancy index and the differences of stresses at peak and residual state for dense sample regardless of confining pressure may be geometric in nature.

REFERENCES

Antony, S. J. and Kruyt, N. P., "Role of interparticle friction and particle-scale elasticity in the shear-strength mechanism of three-dimensional granular media", Phys. Rev. E, (2009), Vol. 79.

Barreto, D. and O'Sullivan, C., "The influence of inter-particle friction and the intermediate stress ratio on soil response under generalized stress conditions", Granular Matter, (2012), Vol. 14, pp. 505-521.

Bishop, A.W, "Correspondence on shear characteristics of a saturated silt measured in triaxial compression", Geotechnique, (1954), Vol. 4(1), pp. 43-45.

Cundall, P. A. and Strack, O. D. L., "A discrete numerical model for granular assemblies", Geotechnique, (1979), Vol. 29(1), pp. 47-65.

Hu, N. and Molinari, J. F., "Shear bands in dense metallic granular materials", J. Mech. Phy. Solids, (2004), Vol. 52(3), pp. 499-531.

Koziki, J. and Donze, F. V., "A new open-source software developed for numerical simulations using discrete modeling methods", Comp. Methods in Applied Mech. Eng., (2008), Vol. 197(49), pp. 4429-4443.

Kruyt, N. P. and Rothenburg, L., "Shear strength, dilatancy, energy and dissipation in quasi-static deformation of granular materials", J. Stat. Mech., (2006), pp. P07021.

Powrie, W., Ni, Q., Harkness, R. M. and Zhang, X., "Numerical modeling of plain strain tests on sands using a particulate approach", Geotechnique, (2005), Vol. 55(4), pp. 297-306.

Rothenburg, L and Kruyt, N. P., "Critical state and evolution of coordination number in simulated granular materials", J. Solids and Structures, (2004), Vol. 41(21), pp. 5763-5774.

Sazzad, M. M. and Islam, M. S., "Macro and micro mechanical responses of granular material under varying interparticle friction", Journal of Civil Engineering, (2008), Vol. 36(2), pp. 87-96.

Sazzad, M. M. and Suzuki, K., "Effect of inter particle friction on the cyclic behavior of granular materials using 2D DEM", J. Geotech. Geoenvir. Eng., ASCE, (2010), Vol. 137, pp. 545-549.

Skinner, A. E., "A note on the influence of interparticle friction on the shearing strength of a random assembly of spherical particles", Geotechnique, (1969), Vol. 19, pp. 150-157.

Suiker, A. S. J. and Fleck, N. A., "Frictional collapse of granular assemblies", J. Applied Mechanics, (2004), Vol. 71(3), pp. 350-358,

Thornton, C., "Numerical simulations of deviatoric shear deformation of granular media", Geotechnique, (2000), Vol. 50(1), pp. 43-53.