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APPROPRIATE AGGREGATION FUNCTION FOR ESTIMATING OF POLLUTION INDEX OF A SELECTED RIVER IN BANGLADESH

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ABSTRACT

The water quality can be studied scientifically if an accurate estimation of water quality is available in the form of an index. Many attempts were made to represent the acceptable ways of water quality using river pollution indices (RPI). The present study aims to select the most appropriate aggregation function for estimating of Bhairav River Pollution Index (BRPI) in the southwestern region of Bangladesh. Following the Delphi technique, a list of 16 pollutant parameters of water samples collected from the selected ten stations of Bhairav river in terms of physical, chemical and heavy metals has been used for possible inclusion in the formulation of BRPI. In the present study, sub-index scores 'p_i' were evaluated for water parameters from the sub-index curves based on their corresponding concentrations. Moreover, the selected variables along with their significance levels, pollutant weight (w_i), concentration (c_i) and sub-index scores (p_i) of pollutant were also considered for formulating of BRPI. In the present study, for evaluating BRPI, 12 aggregate functions were screened out on the basis of ambiguity, eclipsing, constant functional behavior and non accountability of weight. In addition, 6 aggregation functions were also subjected to the analysis of sensitivity. Result revealed that the weighted arithmetic mean aggregation function was found to be the most suitable aggregation function for the estimation of BRPI.

Keywords: *Delphi technique, Aggregation function; Pollution index; Sub-index score, Khulna city.*

1. INTRODUCTION

From the manual data processing of large number of analytical prevention and the faster interpretation of results so that many attempts will make to represent them in more understandable and acceptable ways using water quality indices (WQI) or using river pollution indices (RPI) (Singh *et al*., 2008; Bhargava, 1985). It has been estimated that river water in a fairly wet climate will produce hazardous chemicals such as lead in high concentrations above the drinking water standards for several thousand of years. The state regulatory authorities, in almost all the countries of the world, have framed regulations to safe guard against the contamination of groundwater and surface water sources. The evaluation of water quality in developing countries has become a critical issue in recent years, especially due to the concern of fresh water will be a scarce source in the future (House, 1996). However, the overall composite water quality is sometimes difficult to evaluate from a large number of quality variables.

To describe the water quality based on the pollution load, it is useful to employ a sub-index score of a quality variable to express the individual pollution level of water bodies on a '0' (best quality nil pollution) to '100' (worst quality severe pollution) scale. In order to evaluate the changes of water quality in Bhairav River (in Khulna city, Bangladesh) due to the combined influence of several water quality variables, it was deemed necessary to identify a suitable indexing system. A technique to quantify the pollution potential of River Bhairav in Bangladesh on a comparative scale is developed using an appropriate aggregate water pollution index known as the Bhairav River pollution index (BRPI). BRPI is a single number that expresses the water pollution condition of River Bhairav by integrating measurements of 16 water quality variables and provide a simple and concise method for expressing the water quality of River Bhairav for general uses. Although, the aggregate indices describe the water quality by accounting the impact of various quality variables, the public interest is based entirely on the aesthetic aspects of a water environment (House, 1996).

Over the last three decades, the number of mathematical functions for aggregation of water quality and water pollution indices has been suggested. However, several of the mathematical forms give misleading results in certain circumstances. There were four basic steps primarily involved in the water pollution index design: (1) selection of key water pollutant variables; (2) determination of weight for each selected variables; (3) formulation of sub-indices curves or their mathematical functions; and (4) aggregation of the sub-indices to yield an overall aggregate index. Among these, the aggregation process is the most

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important step and hence, the present study aims to search a suitable aggregation function for BRPI among various available mathematical functions in the literature.

From the research work it has recently mentioned that most aggregation methods suffer from three shortcomings: ambiguity, eclipsing and rigidity. Ambiguity (overestimation) problems exist where the aggregate index is too high and crosses a critical level. The ambiguity free aggregation function is the one in which, if all but one variable, is of acceptable quality. The acceptable quality variable should not influence the aggregation process and the aggregation should reflect the sub-index of the impaired quality variable (Ott, 1978; Swamee and Tyagi, 2007). Eclipsing (underestimation) problems exist when the aggregate index fails to reflect poor water quality of one or more water quality variables (Swamee and Tyagi, 2000).The most suitable aggregation function is the one, which is either free from these problems or it should minimize these effects in the aggregate index. Besides, some aggregation functions are insensitive and exhibit a constant nature with respect to the variation of sub-indices of water pollutant variables, e.g., minimum and maximum operator functions, etc. Further, all the variables do not possess equal significance with regard to the contribution toward water pollution. Also, the aggregation function should possess a high sensitivity to the changes of sub-indices scores of the selected water pollutant variables.

As previously mentioned, 16 pollutant variables were monitored fortnightly for a consecutive period of two days at 10 different sampling stations between Fultala bus-stand and Maniktala, covering a stretch of about 12 km of the course of Bhairav river in Khulna region of Bangladesh. The methodology based on Delphi technique in the formulation of sub-indices and subsequently the BRPI is described. The aggregated BRPI have been computed using 18 different aggregation functions available from the literature. The various functions are subjected to different short-listing criteria such as ambiguity and eclipsing criteria, constant functional behavior, and non-accountability of weight in functions criteria followed by a sensitivity analysis of prescreened functions.

2. Materials and Methods

A total numbers of ten monitoring station were selected between the Fultala busstand and Maniktala along the course of Bhairav River in Khulna city of Bangladesh. The water sampling station 1 (Fultala bus stand) was located about 9.5 km upstream of the Fulbarigate (selected station 8 in Fulbarigate, one of the most congested points in Bhairav River) and station 2 (Damodar Purpopara) was located about 8.5 km upstream of Fulbarigate. Moreover, the station 3 (Damodar), station 4 (Amtala Ghat), station 5 (Atra Afil Mills), station 6 (Munzir Ahmed Road, Maomdanga) and station 7 (Badamtala) were located about 7.5, 6.5, 5, 4, and 2 km, respectively, upstream of Fulbarigate. In addition, station 8 was in Fulbarigate, station 9 (Senpara) was located about 1 km downstream and station 10 (Maniktala) was located about 2 km downstream of the station 8 shown in Figure 1. All the sampling stations were selected on the basis of uses for different purposes. These are the busiest points where the people used for receiving and sending goods. Local people used the station for the departure from one place to another. So there were huge possibilities for the pollution of water from the Launch, goods carrying troller etc. Sampling station 8 was significantly polluted from the wastes discharge from the mills located near the bank of the Bhairav river.

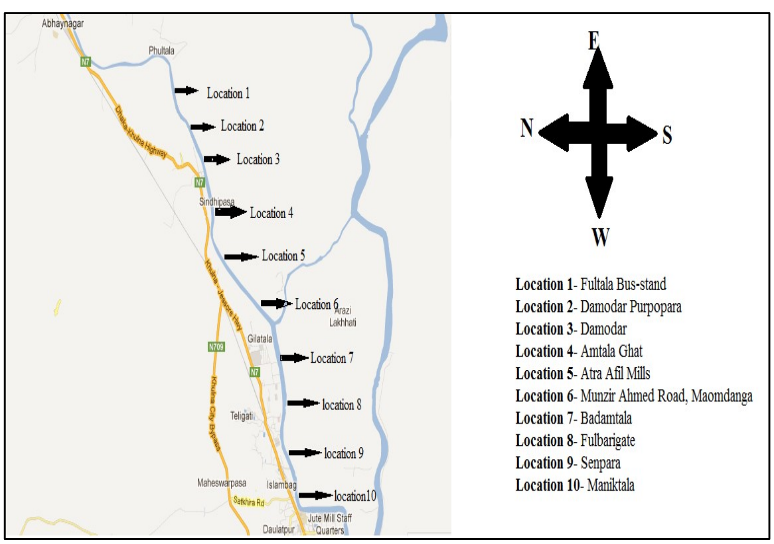


Figure 1: Water sampling stations on the course of river Bhairav in Khulna region of Bangladesh.

2.1 Laboratory Investigations

The water samples were collected during two days period, and all the selected parameters were measured and monitored fortnightly except the heavy metals, which were monitored quarterly. The samples were collected simultaneously from all the selected 10 stations near the bank of the river.. The required 16 parameters for BRPI were analyzed as per the procedure laid down in standard methods (APHA, 1998). Chloride by potentiometric titration method using silver nitrate solution, Alkalinity by titration method, Hardness by EDTA titrimetric method as per the standard method (APHA, 1998). In addition, pH and turbidity were measured by pH meter (HACH, Model No. Sens ion 156) and turbidity meter, respectively. Moreover, Total Solid (TS) dried at 103-105^{0}C was determined in the laboratory as well as Dissolve Oxygen (DO),Biochemical Oxygen Demand (BOD_5) were determined by BOD meter (HACH, Model No. HQ40d) and total Kjeldahl nitrogen (TKN) by macro-Kjeldahl method as per the standard method (APHA, 1998). Total Phosphorus was measured using calibration curve, Sulfate (SO_4^{2-}) by Sulfa Ver 4 method and Cupper (Cu) using spectrophotometer (HACH, DR/2500). In contrary, Lead (Pb) and Chromium (Cr) were measured by using Flame Atomic Absorption Spectrometry (FAAS) Standard Solution pre-concentration period for 7 days.

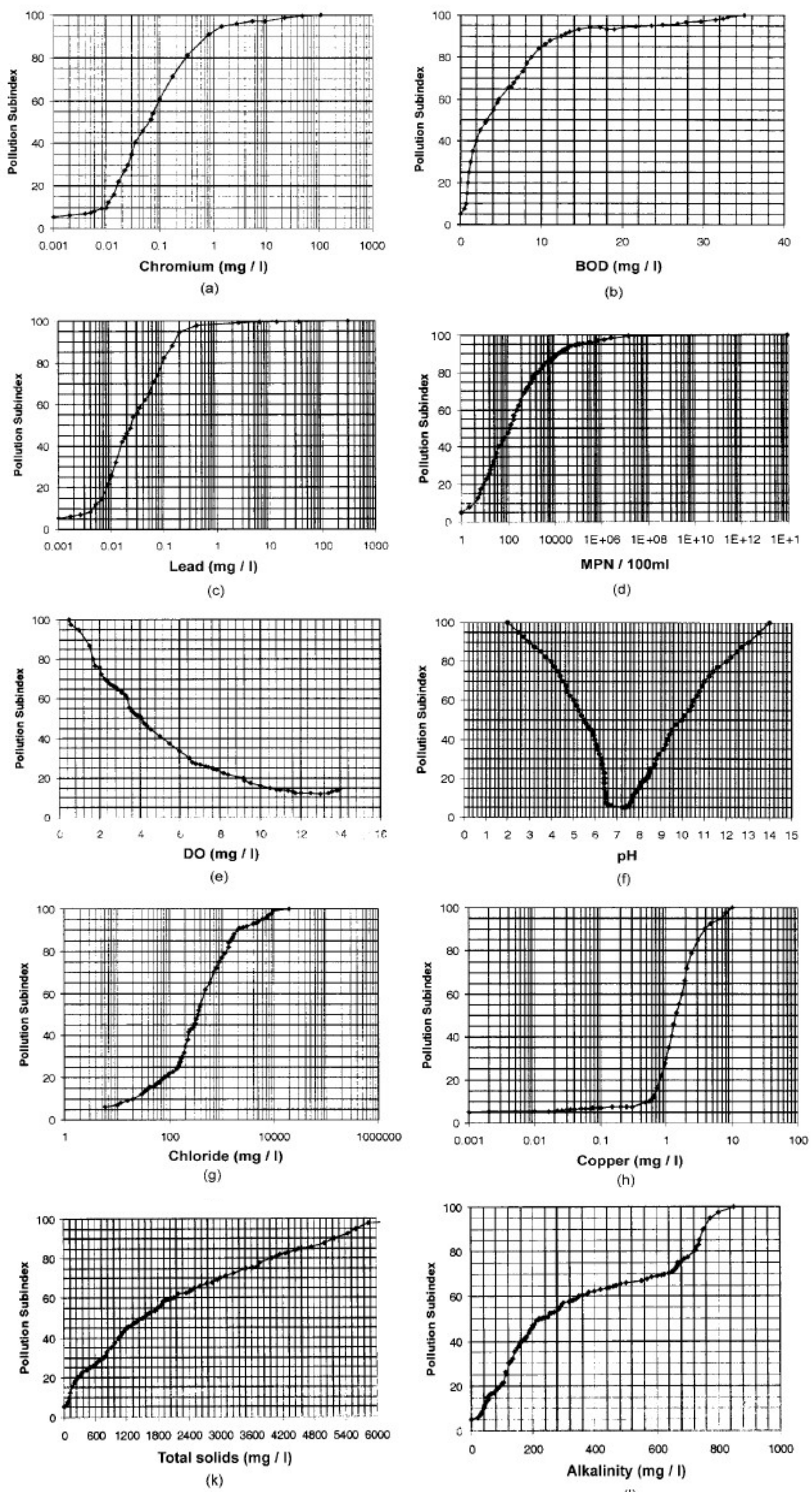
2.2 Methodology for the Formulation of River Pollution Index

The detailed description about the theory and the development of sub-indices based on Delphi Technique in accordance with the technical aspects is available in the literature (Ott, 1978; Smith, 1990; Kumar and Alappat, 2004). The primary steps followed in the formulation of BRPI were briefly summarized here. This technique was briefly described about the possibility of preparing a tool in the form of BRPI, and its subsequent applications in quality description of a selected river for different uses. In addition, the variable according to their increasing significance level on a scale from 1 to 5 was also highlighted here.

Different water quality variables possess different significance levels to overall water quality at different times and locations (Bhargava, 1983; Cude, 2001). The choice of weights is often a source of controversy too. A list of 16 selected variables along with their significance levels is presented in Table 1. As all the pollutant variables received different significance levels, the variables must have different weights. For deriving the weights, the arithmetic sum of the significance ratings for all the selected variables was calculated and each variable was given a weight in proportion to the significance it obtained on a scale of 1, so that the total weight of all the pollutant variables is unity.

To develop rating curves for all 16 selected variables on marked graph sheets with levels of river water pollution (sub-index score) from 0 to 100 indicated on the ordinate of each graph, whereas various level of concentrations of the particular variable, up to the maximum limits reported in literature, were marked along the abscissa. The graph is drawn, which represented the river water pollution produced by the various concentrations of each pollutant variable. The theoretical range of sub-index rating is selected from 5 to 100 shown in Figure 2.

The resulting sub-index rating curves are shown in Figures 2 (a–p) and in which the bold line shows the average sub-index score of each pollutant. The weight factor (w_i) for each pollutant variable, average concentration (c_i) of variables and the corresponding sub-index scores (p_i) evaluated from the rating curves for 5 different sampling stations (1, 3, 5, 8 and 10) are presented in Table 1. Stations were selected on the basis of possibility of maximum pollution. These values were used in the computation of BRPI using different aggregation functions. As all the rating curves were obtained from the expertise and judgment of the specialists, they are implicit nonlinear functions for which no direct mathematical function is available in the literature except for a few variables. Also, the nature and functional behavior of the sub-index curves are not similar for all the selected variables. However, Swamee and Tyagi, 2000 developed some mathematical relationships for uniformly decreasing, non-uniformly decreasing, and unimodal sub-indices based on the nature of sub-index rating curves. But, such relationships have serious limitations due to uncertainties in accurate assessment of the various constants of the function.


Pollution Subindex
Chromium (mg / l)
(a)
BOD (mg / l)
(b)
Lead (mg / l)
(c)
MPN / 100ml
(d)
DO (mg / l)
(e)
pH
(f)
Chloride (mg / l)
(g)
Copper (mg / l)
(h)
Total solids (mg / l)
(k)
Alkalinity (mg / l)
(l)

(Continued)

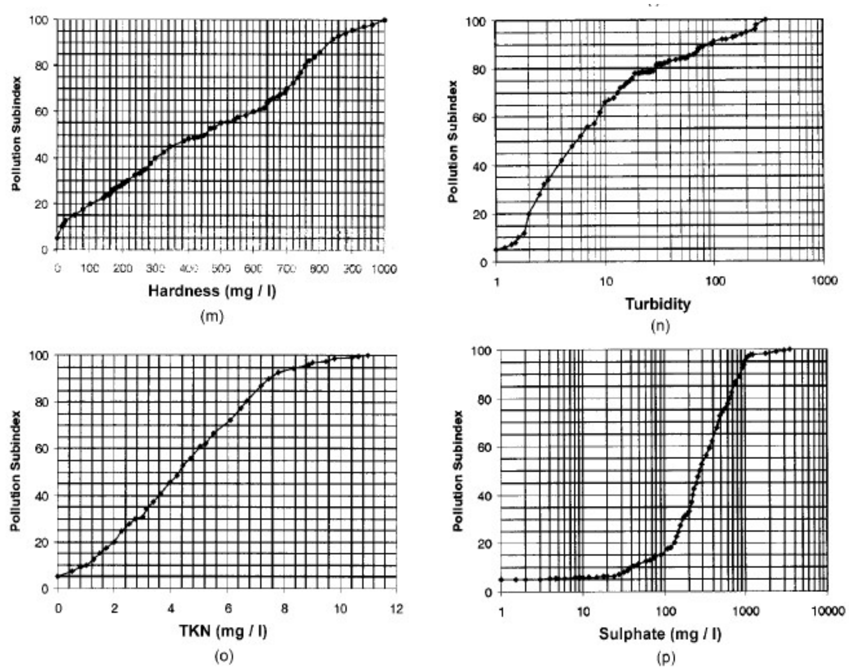


Figure 2: Average sub-index curves for river pollutant variables: (a) chromium (Cr); (b) biochemical oxygen demand (BOD_5); (c) lead (Pb); (d) MPN/100 mL; (e) dissolved oxygen (DO); (f) pH; (g) chloride (Cl^-); (h) copper (Cu); (i) total phosphorus (TPhos) and); (j) zinc (Zn); (k) total solids (TS); (l) alkalinity; (m) hardness; (n) turbidity (NTU); (o) total Kjeldahl nitrogen (TKN); and (p) sulfate (SO_4^{-2})*(after Singh et al. 2008).*

Table 1: Significance level, weight factor, average concentration, and sub-index score of pollutant variables at different monitoring stations

Parameters	Significance	Weight factor	Station 1		Station 3		Station 5		Station 8		Station 10	
		w_i	c_i	p_i	c_i	p_i	c_i	p_i	c_i	p_i	c_i	p_i
BOD_5	4.15	0.077	1.12	25	0.5	8	0.91	18	1.18	24	1.73	35
Cl^-	4.05	0.063	150	24	150	24	150	24	100	22	200	32
pH	4.05	0.072	7.67	7.5	7.72	7.5	7.91	14	7.83	12	6.66	6
DO	4.00	0.074	6.67	28	6.54	30	6.49	30	6.65	28	2.72	65
TS	3.90	0.057	140	12	110	9.5	50	7	120	10	350	28
Alkalinity	3.80	0.053	100	20	80	18	75	17.5	75	17.5	70	17
Hardness	3.30	0.05	167	26	204	28	185	27	167	26	204	28
Turbidity	3.15	0.049	61.3	83	234	95	125	92	226	94.5	117	91
SO_4^{-2}	3.10	0.047	8	5.5	7	5	8	5.5	5	5	12	6
Cr	3.05	0.079	0.04	42	0.03	35	0.04	42	0.05	39	0.05	46
MPN	3.00	0.076	32	22	59	35	45	30	91	45	62	36
Cu	2.80	0.06	0.8	16	0.77	15	1	28	0.82	17	0.95	25
TP	2.65	0.059	0.28	12	0.35	16	0.4	20	0.22	11	0.32	14
Zn	2.60	0.058	0.12	6	0.22	9	0.24	8.5	0.14	7.5	0.22	8
TKN	2.55	0.049	1.05	10	0.95	9.5	0.89	9.5	0.82	9	1.18	12
Pb	2.45	0.077	0.02	35	0.02	44	0.02	46	0.02	36	0.03	53
Total	52.60	1.000										

Note: BOD_5= Biochemical oxygen demand, Cl^-= Chloride, DO= Dissolve oxygen, TS=Total solid, SO_4^{2-} =Sulfate, Cr=Chromium, MPN= most probable number, Cu=Copper, TP= Total phosphorus, Zn=Zinc, TKN= Total Kjeldahl nitrogen, Pb=Lead. w_i= weight for ith parameter; c_i=average concentration or value for ith parameter and p_i= pollution sub-index for ith parameter. All units are in mg/L, except pH (unit less), turbidity (NTU) and PMN/ 100 mL.

2.3 Aggregation Function

Aggregation has been defined as "the process of adding variables or units with similar properties to come up with a single number that represents the approximate overall value of its individual component" (Kumar and Alappat 2004). Aggregation functions can be of additive, multiplicative, minimum or maximum operator forms (Ott 1978; Jollands *et al.* 2003). Like water quality indices, water pollution indices are independent of their functional forms and use all the three forms of aggregation functions (Ott 1978). A number of aggregation function used by different investigators for the description of water quality or water pollution indices were collected from literature. Lists of various aggregation functions with required expression are summarized in Table 2.

Table 2: Aggregation used by different researchers for water quality and pollution indices

No.	Aggregation Function	Function Expression
1	Unweighted arithmetic mean function	$BRPI_{uwa}=1/n\sum^{n}_{i=1}p_i$
2	Weighted arithmetic mean function	$BRPI_{wa}=\sum^{n}_{i=1} w_ip_i$
3	Root sum power function	$BRPI_{rs}p=(\sum^{n}_{i=1}p^{r}_{i})^{1/r}$
3a	Root sum power function (r=2)	$BRPI_{r2sp}=(\sum^{n}_{i=1}p^{2}_{i})^{1/2}$
3b	Root sum power function (r=4)	$BRPI_{r4sp}=(\sum^{n}_{i=1}p^{4}_{i})^{1/4}$
3c	Root sum power function (r=10)	$BRPI_{r10sp}=(\sum^{n}_{i=1}p^{10}_{i})^{1/10}$
4	Weighted root sum power function	$BRPI_{wrsp}=(\sum^{n}_{i=1}w_ip^{r}_{i})^{1/r}$
4a	Weighted root sum power function (r=4)	$BRPI_{wr4sp}=(\sum^{n}_{i=1}w_ip^{4}_{i})^{1/4}$
4b	Weighted root sum power function (r=10)	$BRPI_{wr10sp}=(\sum^{n}_{i=1}w_ip^{10}_{i})^{1/10}$
5	Root-mean-square function	$BRPI_{rms}=(1/n\sum^{n}_{i=1}p^{2}_{i})^{1/2}$
6	Weighted root sum square function	$BRPI_{wrss}=(1/n\sum^{n}_{i=1}w_ip^{2}_{i})^{1/2}/\sum^{n}_{i=1} wi$
7	Maximum operator function	$BRPI_{max}=\max(p_1,p_2,p_3-p_n)$
7a	Minimum operator function	$BRPI_{min}=\min(p_1,p_2,p_3-p_n)$
8	Unweighted ambiguity and eclipsity free function (*r*=0.4)	$BRPI_{uw0.4aef}=(\sum^{n}_{i=1}P^{2.5}_{i})^{0.4}$
9	Weighted ambiguity and eclipsity free function (*r*=0.4)	$BRPI_{w0.4aef}=(\sum^{n}_{i=1}w_ip^{2.5}_{i})^{0.4}$
10	Weighted average concentration function	$BRPI_{wac}=k\sum^{n}_{i=1}p_ic_i/\sum^{n}_{i=1}c_i$
11	Sub-index powered weight function	$BRPI_{spw}=(\sum^{n}_{i=1}p^{w}i_{i)}$
12	Unweighted multiplicative function	$BRPI_{uwm}=(\prod^{n}_{i=1}p_i)^{1/n}$
13	Weighted multiplicative/weighted geometric mean function	$BRPI_{wm}=\prod^{n}_{i=1}p^{wi}_{i}$
14	Square root unweighted harmonic mean square function	$BRPI_{sruwh}=\sqrt{}(n/\sum^{n}_{i=1}1/p^{2}_{i})$

2.4 Criteria for Selection of Suitable Aggregation Function

The following aspects/criteria need to be considered in the selection of an appropriate aggregation function for the estimation of an aggregate index. These criteria are gleaned/judged from the literature (Swamee and Tyagi 2000; Kumar and Alappat 2004; Swamee and Tyagi 2007).

a) The most appropriate aggregation function is the one that is either free from or minimizes the overestimation (ambiguity), underestimation (eclipsing) and rigidity problems. Overestimation (ambiguity) problems arise when the aggregate index exceeds the critical level without any of the sub-indices exceeding the critical levels. Underestimation (eclipsing) problems exist when the aggregate index is too low and does not exceed the critical level. Rigidity problems arise due to inclusion of additional variables, and the aggregate index shows a low result because of the use of faulty aggregation function, indicating impaired water quality. In the present study, the rigidity problems are not given due to the consideration of variables included are identified by expert opinion.
b) The aggregation function selected for any environmental index shall also meet the following general criteria, i.e., it should
   (1) be sensitive to the changes in an individual variable throughout its range
   (2) not be biased toward good or poor environmental quality
   (3) consider weighting factors, as all variables included in the index are not equal contributors to water pollution
   (4) be relatively easy to use.
c) When competing aggregation functions produce similar results with respect to overestimation and underestimation, the most appropriate aggregation function will be the one that is mathematically simple.

d) An aggregation approach is successful if all assumptions and sources of data are identified, the methodology is transparent and publicly reported, and an index can be readily disaggregated into the separate components with no information lost.

3. RESULTS AND DISCUSSIONS

The most appropriate aggregation function for estimating of BRPI can be primarily analyzed and screened out on the basis of the following criteria.

3.1. Based on Ambiguity Criterion

From the results of BRPI presented in Table 3, it was evident that root sum power ($BRPI_{r2sp}$), fourth root sum power ($BRPI_{r4sp}$), tenth root sum power ($BRPI_{r10sp}$), and unweighted ambiguity and eclipsity free ($BRPI_{uw0.4aef}$) aggregation functions produced ambiguous results and indicating less polluted water. Also, the computed BRPI values exceed the maximum reported individual pollutants sub-index scores. Therefore, they were not considered to be the composite water quality indices. From Table 3, it can be seen that the values of BRPI also exceed the theoretical range of BRPI, i.e., 5–100 except for the $BRPI_{r10sp}$ function. However, the $BRPI_{r10sp}$ function produced the least ambiguous result, followed by the functions of $BRPI_{r4sp}$, $BRPI_{uw0.4aef}$, and $BRPI_{r2sp}$. Although, the function of $BRPI_{r10sp}$ does not show much ambiguity in the results, but it cannot be used for calculating of BRPI value, as its results cannot be used to compare the fine gradations of river water pollutant. On the basis of ambiguity criterion, the computed BRPI values using functions of $BRPI_{r2sp}$, $BRPI_{r4sp}$, $BRPI_{r10sp}$, and $BRPI_{uw0.4aef}$ were found to be ambiguous and hence were short-listed. The computed BRPI values using these four aggregation functions are summarized in Table 4.

3.2. Based on Eclipsing Criterion

The computed values of BRPI using square root unweighted harmonic mean square ($BRPI_{sruwh}$) and sub-index powered weight ($BRPI_{spw}$) functions indicated the high eclipsing of water data in river and hence presented in Table 4 for five sampling stations. From Table 4, it can be seen that the BRPI values were very low at all the sampling stations as compared to the other additive form aggregation functions. Also, it is gleaned from Table 3 that the results of the two multiplicative aggregation functions, i.e., unweighted ($BRPI_{uwm}$) and weighted ($BRPI_{wm}$), were relatively low and exhibited eclipsing problems in comparison to additive functions, such as unweighted ($BRPI_{uwa}$) and weighted ($BRPI_{wa}$) arithmetic mean functions. Although $BRPI_{uwa}$ and $BRPI_{wa}$ also suffers from the eclipsing problems, the eclipsity produced relatively the smaller one as the number of variables included in the aggregation function was large. Therefore, on the basis of the eclipsing criterion, the functions of $BRPI_{sruwh}$ and $BRPI_{spw}$ could be easily ruled out for the estimation of BRPI.

Table 3: Computed BRPI values at different sampling stations

Aggregation	Station 1	Station 3	Station 5	Station 8	Station 10
$BRPI_{uwa}$	23.38	24.28	26.19	25.22	31.38
$BRPI_{wa}$	24.61	27.32	31.53	33.65	42.42
$BRPI_{r2sp}$	119.12	129.93	133.05	131.97	154.71
$BRPI_{r4sp}$	85.87	97.46	95.37	97.32	101.18
$BRPI_{r10sp}$	83.01	95.01	92.01	94.51	91.36
$BRPI_{wr4sp}$	41.05	46.41	45.62	46.45	49.59
$BRPI_{wr10sp}$	61.40	70.27	68.06	69.90	67.71
$BRPI_{rms}$	29.78	32.48	33.26	32.99	38.68
$BRPI_{wrss}$	29.15	31.52	32.61	32.30	39.02
$BRPI_{max}$	83.00	95.00	92.00	94.50	91.00
$BRPI_{min}$	5.50	5.00	5.50	5.00	6.00
$BRPI_{uw0.4aef}$	100.67	111.63	112.24	112.59	127.31
$BRPI_{w0.4aef}$	32.14	35.35	35.88	35.89	41.95
$BRPI_{wac}$	26.27	42.29	35.86	43.50	35.36
$BRPI_{spw}$	19.26	19.23	19.40	19.34	19.61
$BRPI_{uwm}$	20.19	17.68	19.54	21.26	22.95
$BRPI_{wm}$	21.14	18.33	20.17	23.22	21.95
$BRPI_{sruwh}$	11.62	11.28	12.64	12.03	13.01

3.3. Based on Constant Functional Behavior Criterion

The weighted average concentration ($BRPI_{wac}$), maximum operator ($BRPI_{max}$), and minimum operator ($BRPI_{min}$) aggregation functions showed almost a constant functional behavior at a particular sampling station when the sub-index scores are varied from 5 to 100 for both the chromium and sulfate pollutant in water. Thus, the estimation of BRPI using these aggregation functions appeared to be the insensitive to the changes of sub-index scores of the pollutant variables. $BRPI_{max}$ takes the value of the largest of any of the sub-indices. Like $BRPI_{r10sp}$, $BRPI_{max}$ does not show much ambiguity of results, but it cannot be used to compare the micro level concentrations of river water pollutants. Similarly, $BRPI_{min}$ is also an ambiguity and eclipsity free function (Smith 1990; Swamee and Tyagi 2000), but cannot be used to assess the sensitivity of function at the macro level concentrations of pollutant variables. Thus, based on the constant functional behavior criterion, the aggregation functions of $BRPI_{wac}$, $BRPI_{max}$, and $BRPI_{min}$were not suitable for the estimation of BRPI.

3.4. Based on Non-accountability of Weights in Functions Criterion

The number of aggregation function (Table 2) does not account for the weight of pollutant variables. As the weights are derived from the experts' opinions, it must be accounted for the aggregation function for better estimating of composite water quality.The functions of $BRPI_{uwa}$, $BRPI_{rms}$, and $BRPI_{uwm}$ do not consider the weight of variables, and all the variables are assumed to be of weight unity, which appeared to be unrealistic as different pollutant variables have different levels of significance from a water pollution point of view. The function of $BRPI_{uwm}$ also suffers from the eclipsing problem. Thus, due to non accountability of pollutant's weight, the aggregation functions of $BRPI_{uwa}$, $BRPI_{rms}$ and $BRPI_{uwm}$ were also ruled out for the estimation of BRPI as provided in Table 4.

Table 4: Preliminary screening of aggregation functions at different sampling stations

Criteria/screened functions	Station 1	Station 3	Station 5	Station 8	Station 10
Based on ambiguity criteria					
$BRPI_{r2sp}$	119.12	129.93	133.05	131.97	154.71
$BRPI_{r4sp}$	85.87	97.46	95.37	97.32	101.18
$BRPI_{r10sp}$	83.01	95.01	92.01	94.51	91.36
$BRPI_{uw0.4aef}$	100.67	111.63	112.24	112.59	127.31
Based on eclipsing criteria					
$BRPI_{spw}$	19.26	19.23	19.40	19.34	19.61
$BRPI_{sruwh}$	11.62	11.28	12.64	12.03	13.01
Based on constant functional behavior					
$BRPI_{min}$	5.50	5.00	5.50	5.00	6.00
$BRPI_{max}$	83.00	95.00	92.00	94.50	91.00
$BRPI_{wac}$	26.27	42.29	35.86	43.50	35.36
Based on non accountability of weight in functions					
$BRPI_{rms}$	29.78	32.48	33.26	32.99	38.68
$BRPI_{uwa}$	23.38	24.28	26.19	25.22	31.38
$BRPI_{uwm}$	20.19	17.68	19.54	21.26	22.95

3.5. Sensitivity Analysis

The six aggregation functions of $BRPI_{wa}$, $BRPI_{wr4sp}$, $BRPI_{wr10sp}$, $BRPI_{w0.4aef}$, $BRPI_{wrss}$ and $BRPI_{wm}$ were subjected to the analysis of sensitivity to the changes of individual pollutant's sub-index value for the selection of the most suitable aggregation function for estimating of BRPI. All these functions account for the weight of pollutant variables and have relatively less ambiguity and eclipsing problems.

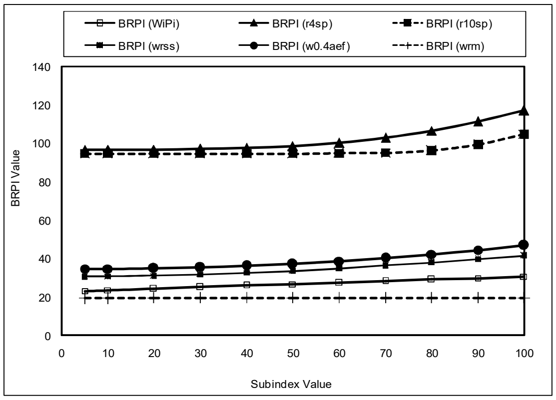


Figure 3: Results of sensitivity analysis with respect to changes of sub-index score of Chromium for station 8.

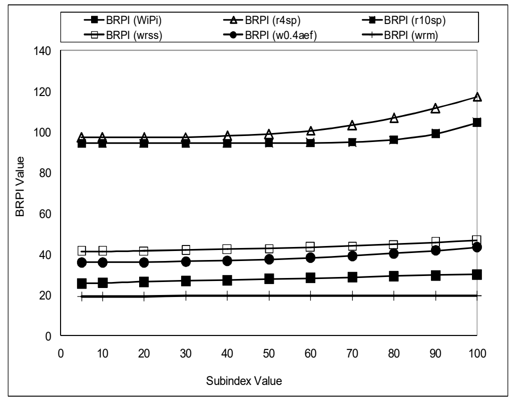


Figure 4: Results of sensitivity analysis with respect to changes of sub-index score of Sulfate for station 8.

For performing of sensitivity analysis, the sub-index scores of chromium and sulfate were varied from 5 to 100 in the same data set for the average concentration of the variables (Table 1) for five selected sampling stations along the course of river. The variation of BRPI values with respect to the changes of sub-index value of chromium and sulfate are plotted in Figs. 3 and 4, respectively, for sampling station 8 (Fulbarigate) only. A similar plot could be observed for the remaining four sampling stations. The computed BRPI values for all the 5 stations (1, 3, 5, 8 and 10) at sub-index values of 5 and 100 for both chromium and sulfates, respectively, can be obtained from graph (Figure 2). Using the BRPI values from Figure 2, the percentage variation of the BRPI values over the minimum value for the sub-index variation (p_i=5) of chromium and sulfate are shown in Table 5 for all the five sampling stations along with their average values.

Table 5: Sensitivity analysis of aggregation for changing of sub-index scores of chromium and sulfate

Aggregation Function	Percent changes aof BRPI for Cr and SO_4^{-2}											
	Station 1		Station 3		Station 5		Station 8		Station 10		Average BRPI	
	Cr	SO_4^{-2}	Cr	SO_4^{-2}	Cr	SO_4^{-2}	Cr	SO_4^{-2}	Cr	SO_4^{-2}	Cr	SO_4^{-2}
$BRPI_{wm}$	27	21.8	41.6	21.0	31.3	15.9	33.2	27.4	27.1	18.5	32.0	20.1
$BRPI_{wa}$	36.1	21.1	34.1	18.2	31.7	16.7	32.6	17.2	25.7	13.5	32.0	17.4
$BRPI_{wrss}$	44.9	24.3	36.8	21.1	35.9	19.8	35.9	20.2	25.6	14.2	35.8	19.9
$BRPI_{w0.4aef}$	46.1	26.3	35.6	21.5	35.9	20.8	35.2	20.7	26.5	14.6	35.9	20.8
$BRPI_{wr4sp}$	41.6	27.3	28.6	18.9	30.7	19.9	29.0	18.8	24.2	15.3	30.8	20.0
$BRPI_{wr10sp}$	27.4	21.6	13.9	9.9	16.7	12.2	14.3	10.3	17.2	12.6	17.9	13.3

From Table 5, it can be seen that the aggregation function $BRPI_{w0.4aef}$ was the most sensitive one in comparison to other aggregation functions and showing an average change in BRPI values of 35.87 and 20.76%, for chromium and sulfate, respectively. The next most sensitive function is $BRPI_{wrss}$ showed an average of 35.83 and 19.92 % variation in BRPI values for the two extreme pollutants, followed by the functions of $BRPI_{wm}$, $BRPI_{wa}$, $BRPI_{wr4sp}$, and $BRPI_{wr10sp}$ in decreasing order of sensitivity.

From the plots in Figures 3 and 4, it is observed that the function of $BRPI_{wm}$ was the almost insensitive in sub-index variation for both the chromium and sulfate. Therefore, this function does not give the better estimation of overall water quality of Bhairav River for changes of sub-index scores. The function of $BRPI_{wr10sp}$ showed a nonlinear behavior to changes of sub-index scores and exhibited a low sensitivity at low sub-index scores ($p_i \leq 70$). The sensitivity improved non-uniformly at a faster rate beyond ($p_i > 70$). In the case of function $BRPI_{wr4sp}$, whose sensitivity was poor for $p_i \leq 40$, improved gradually up to $p_i \leq 60$ and improved rapidly for p_i above 60. These two functions do not reflect truly the fine gradations of water pollutant variables. $BRPI_{w0.4aef}$ and $BRPI_{wrss}$ were being a nonlinear function, both have low sensitivity at low sub-index scores ($p_i \leq 30$) and its sensitivity increased rapidly beyond p_i=40. These functions have not yet been used for aggregation of water quality or water pollution indices. Although the sensitivity analysis showed that the variation of $BRPI_{w0.4aef}$ values for the change of sub-index scores of chromium and sulfate was the highest as it is previously discussed, it suffers from the drawback that the function was nonlinear and showed biased results for higher sub-index scores. Therefore, these aggregation functions showed inconsistent behavior and hence may not be useful for aggregating the water pollution indices especially when the fine gradations of water pollutants are essential.

The aggregation function of $BRPI_{wa}$ showed a uniform and true linear (R^2=1) behavior in relation to the variation of sub-index score for both the chromium and sulfate, and ranks second in sensitivity among the six considered aggregation functions. Also, this function exhibited less eclipsing problem as compared to $BRPI_{w0.4aef}$. Thus, it can be inferred that the weighted arithmetic mean aggregation function ($BRPI_{wa}$) was found to be the most appropriate aggregation function for estimating of BRPI.

4. CONCLUSION

Results revealed that the aggregation functions of $BRPI_{r2sp}$, $BRPI_{r4sp}$, $BRPI_{r10sp}$, and $BRPI_{uw0.4aef}$ produced more ambiguous problems. The functions of $BRPI_{sruwh}$ and $BRPI_{spw}$ exhibited comparatively the higher eclipsing problems. Moreover, the aggregation functions of $BRPI_{wac}$, $BRPI_{max}$, and $BRPI_{min}$ showed almost the constant functional variation at a particular sampling station with the irrespective variation of sub-index scores. Also, the function of $BRPI_{max}$ was ambiguity and eclipsity free, but it could not be used for the estimation of BRPI, as it was least sensitive to fine gradations of the changes in concentrations of water pollutant variables. The functions of $BRPI_{uwa}$, $BRPI_{rms}$, and $BRPI_{uwm}$ did not take into consideration and assumed all the pollutant variables have the same significance level. In addition, the aggregation function of $BRPI_{wa}$ exhibited less eclipsing problems as well as a uniform and true linear behavior with the variations of sub-index scores for both the pollutants ofchromium and sulfate, and ranks second in sensitivity among six aggregation functions. Thus, the weighted arithmetic mean aggregation function was found to be the most suitable aggregation function for estimating of BRPI.

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