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A MODIFIED ALGORITHM FOR SOLVING UNBALANCED TRANSPORTATION PROBLEMS

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ABSTRACT

Least Cost Method (LCM) is a very simple and efficient approach to find out Initial Basic Feasible Solution (IBFS) in the case of balanced Transportation Problems (TP). But, in real life, most of the problems are the unbalanced i.e. amount of supply commodities is not equal to that of demand. In order to apply any transportation algorithm, it is required to make the problem balanced. The existing LCM, as well as other approaches like VAM's algorithm, solves the unbalanced TP by introducing dummy routes. Zero values are assigned as unit transportation cost for each dummy route so that total transportation cost is unaffected due to dummy transportations. In the case of LCM, it frequently falls in a pitfall and consequences the total transportation cost becomes larger. Furthermore, it needs more iteration to find out the optimal solution. In order to escape from this pitfall, the existing LCM algorithm needs to be modified. In this research, a modified algorithm based on LCM is proposed for unbalanced transportation problems to find out an initial basic feasible solution. Several numerical illustrations have been carried out to justify the validity and effectiveness of the proposed algorithms. Moreover, illustrations results are also compared with existing ones namely LCM and VAM to find out the efficiency of the proposed algorithm in the case of unbalanced TP. It is also remarked that the proposed modified algorithm is also suitable for balanced TP. Numerical results reveal that the proposed modified algorithm is much more efficient compared to existing ones in the case of unbalanced TP.

Keywords: *Unbalanced Transportation Problem, Dummy Route, Initial Basic Feasible Solution, Algorithm, Transportation Tableau*

1. INTRODUCTION

A large number of physical problems in the business arena can be modeled as a Transportation problem. One of the most important applications of Transportation Problems (TPs) is products distribution. The aim of this problem is to minimize the cost of shipping goods of a single commodity from numerous origins (supply origins/plants) to different localities (demand destinations/warehouses) so that the needs of each locality are met and every origin operates within its capacity. If the total availability of commodities of all origin is equal to the total amount of demand from all destinations, then the problem is known as balanced TP. Otherwise, it is unbalanced TP.

Numerous approaches are available in the literature and also research works are ongoing to obtain more efficient algorithms to solve TP. Most of the methods are suitable for balanced TP to find IBFS. LCM is the well-known, simple but efficient method for finding IBFS of any balanced TP (Taha, 2003). It can be also implemented in the case of unbalanced TP with some modification of the mathematical model of the problem. Another well-known approach is called VAM (Reinfeld and Vogel, 1958), which is able to find better IBFS for balance TP but it is computationally time-consuming.

Besides LCM and VAM, many researchers proposed several methods for both balanced as well as unbalanced TP. Some of the important related recent works are briefly cited here in order to focus current state-of-arts regarding TP to find out IBFS. Most of the approaches are the variants of VAM obtained by modifying some tricks on cost matrix (Korukoglu and Balli, 2011; Amirul Islam *et al*., 2012; Sudhakar *et al*., 2012; Raigar *et al*., 2017; Juman and Hoque, 2015; Juman and Nawarathne, 2019; Prajwal *et al*., 2019) for solving balanced TP. Recently, Hosseini (2017) and Amaliah *et al.* (2019) developed two different methods named Total Differences Method 1 (TDM1) and Total Opportunity Cost Matrix Minimal Total (TOCM-MT) to obtain IBFS of balanced TP.

Though huge numbers of research articles are available in literature on balanced TP but relatively fewer papers are available regarding unbalanced TP. A modification (SVAM) was proposed by Shimshak (1981) in which any penalty that involves dummy routes were ignored. Goyal (1984) proposed a simple rule for improving VAM for unbalanced transportation problems with the replacement of zero costs in the dummy column by the largest unit transportation costs. Ramakrishnan, (1988) improved Goyal's modified VAM for the unbalanced transportation problem by subtracting or adding suitable constraints to the rows and columns of the cost matrix. Balakrishnan (1990) suggested a further modification of Goyal's modified VAM approach. In their approach, all the column penalties are computed as Goyal's modified VAM, except for the dummy column and for the rows, compute the penalties by calculating the difference between the lowest cost and the second-lowest cost but ignoring the dummy column. Kulkarni and Datar (2010) developed a heuristic based algorithm to find out IBFS to minimize transportation cost for unbalanced TP. Girmay and Sharma (2013) also proposed another heuristic

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approach to find out IBFS for both balanced and unbalanced TP. Juman *et al*. (2013) performed a sensitivity analysis on VAM procedure to see the effect of balancing and unbalancing issues on the initial cost of VAM. Recently, Geetha *et al.* (2018) proposed Standard Deviation Method (SDM) for finding an IBFS for unbalanced TP and also claimed that the SDM provides comparatively better IBFS result than that of VAM. It is observed that most of the approaches available in the literature are actually someway variants of VAM's method. Very recently Jamali and Reza (2019) show some elementary experimental results on LCM in which dummy transportation costs are set non-zero.

To find the IBFS, though there are many methods existing in the literature, there is always eagerness among the researchers to find a method which can give a solution that is either optimal or at least converges quicker to the optimal solution. In this paper, a modified LCM algorithm is proposed which is efficient for both balanced and unbalanced TP. To overcome the shortcoming of LCM regarding unbalanced TP, we have set a non-zero amount of transportation cost to dummy routes in a convenient way so that total transportation cost does not affect on total transportation cost.

This article is organized as follows: After an introduction and a brief literature review, mathematical model of TP which are given in section one and two respectively. Then, the existing algorithm with the mathematical model of unbalanced TP and proposed algorithm are discussed in section three and four respectively. A detail experimental study with a numerical instance is presented in section five. Comparison studies among several approaches along with proposed approaches are discussed in section six. The conclusion includes in section seven.

2. MATHEMATICAL MODEL OF TRANSPORTATION PROBLEM

In general, the Transportation Problem (TP) can be designed as a Linear Programming (LP) problem as follows: Let, the number of supply commodities at origin i is a_i and the amount of demand commodities at destination j is b_j. The cost of transporting one unit from origin i to destination j is c_{ij}. It is obvious that, $a_i \geq 0$ for each i and $b_j \geq 0$ for each j. Let, x_{ij} be the amount of quantity transported from origin i to destination j. Then, $c_{ij}x_{ij}$ (cost × quantity) be the amount of cost to transport x_{ij} commodities along the route r_{ij} from origin i to destination j. Therefore, the total cost for the of transportation of the commodities from all origins to all destinations is given by

$$\mathbf{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \tag{1}$$

where, m is the total number of origins and n is the total number of destinations.

The objective of the problem is to minimize the total transportation cost subject to the given constraints. The above problem can be modeled as LP given below:

$$\text{Minimize, } \mathbf{Z} = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \qquad \text{(Total transportation cost)} \tag{2}$$

Subject to,

$$\sum_{j=1}^{n} x_{ij} = a_i\,, i = 1,2,\cdots,m \qquad \text{(Supplies at origin)}$$

$$\sum_{i=1}^{m} x_{ij} = b_j\,, j = 1,2,\cdots,n \qquad \text{(Demands at destination)}$$

$$x_{ij} \geq 0 \quad \forall i,j \qquad \text{(Quantities)}$$

The above problem will be called unbalanced TP if the amount of supply commodities is not equal to demand commodities i.e., $\sum_i^m a_i \neq \sum_j^n b_j$. Any LP approach, like the Simplex method, is able to solve the problems. But it is known that the Simplex method is very time consuming and tedious method. Moreover, it needs an Initial Basic Feasible Solution (IBFS) too. On the other hand, if the amount of supply commodities is equal to the amount of demand commodities i.e., $\sum_i^m a_i = \sum_j^n b_j$, then above problem is treated as balanced TP. For the existence of the special structure of balanced TP, it can be solved in a more convenient way by introducing Transportation Tableau.

3. LCM ALGORITHM FOR UNBALANCED TP

In the case of unbalanced TP, the existing LCM approach introduces dummy routes to make the problem balanced. Inconsequence, the number of total routes is increased either due to introducing a dummy origin or due to introducing dummy destination. As a result, the total transportation cost will be affected due to these dummy transportation costs. So, in order to escape from these dummy transportation costs, the existing LCM sets zero transportation cost corresponding to each dummy route. The main steps of LCM for unbalanced TP are given below:

Step 1: Find out the excessive amount of commodities and identify whether it is an origin or sink and create a dummy origin or destination accordingly:

(a) If $\sum_i^m a_i - \sum_j^n b_j = c > 0$ then introduce a dummy destination such that $b_{n+1} = c,$ so $\sum_i^m a_i = \sum_j^{n+1} b_j$ and set $p = m;\ q = n + 1$.

(b) If $\sum_i^m a_i - \sum_j^n b_j = c < 0$ then introduce a dummy origin such that $a_{m+1} = c,$ so $\sum_i^{m+1} a_i = \sum_j^n b_j$ and set $p = m + 1\ ;\ q = n$.

(c) Now the problem becomes balanced TP. So the mathematical model of modified TP is given as follow:

Minimize, $\mathbf{Z} = \sum_{i=1}^p \sum_{j=1}^q c_{ij}\, x_{ij}$ (Total transportation cost) (3)

Subject to,

$\sum_{j=1}^p x_{ij} = a_i\, , i = 1,2, \cdots, p$ (Supplies at origin)

$\sum_{i=1}^q x_{ij} = b_j\, , j = 1,2, \cdots, q$ (Demands at destination)

$x_{ij} \geq 0 \quad \forall i, j$ (Quantities)

Now it is obvious that, $\sum_i^p a_i = \sum_j^q b_j$ (Balanced condition)

Note that the number of equations (p+q-1), as well the number of decision variables (*pq*), is increased due to introducing dummy routes.

4. PROPOSED MODIFIED LCM ALGORITHM

Since according to LCM, the rule of allocation flow is – the route with least transportation cost allocated first, so in the existing LCM algorithm allocates all possible amount of commodities to dummy routes first as its transportation cost is minimum namely zero. As a result, a good amount of commodities bounds to allocate to the routes which correspond to high transportation costs. In order to escape from such pitfall, we consider a non-zero value namely sum of all existing transportation cost correspond to each dummy route so that algorithm bound to away from these dummy routes in starting allocation stages. But due to non-zero transportation cost, the total transportation must is affected by these dummy transportations. Therefore we need to modify the existing LCM algorithm so that it is able to solve both balanced and unbalanced TP so that total cost is unaffected due to dummy routes if any.

4.1 Reformulation of mathematical model (Step 1)

It is known that a new dummy origin/destination is introduced in order to make an unbalanced TP to balanced TP. As a result, the number of origins/destinations increased by one. But, whatever be the unit transportation cost corresponding to the dummy node, the total transportation cost should be unaffected regarding dummy transportation through dummy routes as it is mimic. But it is just needed for mathematical modeling. By exploiting these observable facts, the mathematical model of unbalanced TP is formulated as follows:

At first proposed algorithm examines whether the TP is balanced or not. That is to find out the surpass amount of commodities if any and identify whether it is an origin or destination. If there exist an exceed amount of commodities then create a dummy origin or destination where necessary. Formally:

a. If $\sum_i^m a_i - \sum_j^n b_j = c > 0$ then launch a dummy destination such that, $b_{n+1} = c,$ so $\sum_i^m a_i = \sum_j^{n+1} b_j$ and set $p = m;\ q = n + 1$.

b. If $\sum_i^m a_i - \sum_j^n b_j = c < 0$ then launch a dummy origin such that, $a_{m+1} = c,$ so $\sum_i^{m+1} a_i = \sum_j^n b_j$ and set $p = m + 1\ ;\ q = n$.

c. If $\sum_i^m a_i - \sum_j^n b_j = 0$ then problem is balanced, so set $p = m;\ q = n$.

After the above modification, the mathematical model of the modified TP is as follow:

Minimize, $\mathbf{Z} = \sum_{i=1}^m \sum_{j=1}^n c_{ij}\, x_{ij}$ (Total transportation cost) (4)

Subject to,

$\sum_{j=1}^p x_{ij} = a_i\, , i = 1,2, \cdots, p$ (Supplies at origin)

$\sum_{i=1}^q x_{ij} = b_j\, , j = 1,2, \cdots, q$ (Demands at destination)

$x_{ij} \geq 0 \quad \forall i, j$ (Quantities)

and obviously, $\sum_i^p a_i = \sum_j^q b_j$ (Balanced condition)

It is remarked that for the calculation of total transportation cost, the algorithm is able to ignore the costs due to dummy transportation. So whatever be the unit dummy transportation cost to each dummy route the total transportation cost is unaffected regarding the dummy transportations.

4.2 Formulation of the unit cost of dummy route (Step 2)

If the given problem is unbalanced TP so that we have to introduce dummy routes to make the problem balanced. Our next task is to set the amount of unit transportation cost for each dummy route (cell). It is defined in such a way that these routes (route corresponding to dummy node) have the largest transportation cost but at the same time, it should be generic. Therefore, for each and every dummy routes we set unit transportation cost

$$T_c = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} \tag{5}$$

4.3 Allocation procedure of the Proposed Modified LCM Algorithm (Step 3)

The allocation procedures of the proposed Modified LCM (MLCM) are identical to existing LCM and which is very simple and easy to implement. The algorithm prefers the smallest cost first for allocation and so on. That is, allocate the number of commodities to the route (cell) corresponding to minimum transportation cost. But if there are more than one routes have identical transportation cost then algorithm allocates to the route in which a larger amount of commodity is available for allocation. Again if more than one routes have same transportation cost as well available amount of commodities to be allocated are identical (i.e. the min $\{S_i, D_j\}$ values for two routes are identical) then break the tie arbitrarily. It is noted that after allocation to the cell C_{ij} contained the minimum transportation cost, the cell C_{ij} will be exhausted along with its either i th row or j th column which contains minimum commodities. Formally

(a) If $S_i = \min\{S_i, D_j\}$ then block all routes corresponding to origin i (row i) and readjust availability of destination j: $D_j = D_j - S_i$.
(b) Or if $D_j = \min\{S_i, D_j\}$ then block all routes corresponding to destination j (column j) and readjust availability of destination i: $S_i = S_i - D_j$.
(c) Or $S_i = D_j = \min\{S_i, D_j\}$ then block all routes corresponding one of them (origin i or destination j) arbitrarily and readjust availability of destination or origin accordingly.

So for further allocation if any we need to consider reduced matrix.

Step 4 (Termination): Continuing the allocation procedure of Step 3 sequentially until all possible allocations are done.

5. DETAIL EXPERIMENTATION FOR A TYPICAL INSTANCE

For the justification and effectiveness of the proposed Modified Least Cost Method (MLCM), here we have considered a typical example:

Example 1: Four manufacturers (S) have produced 170, 250, 130 and 350 units of bottled water respectively which will be distributed to three markets (D) with demands 200, 300, and 500 units. Each transportation process has an associated cost represented in Table 1.

Table 1: Transportation Problem

	D_1	D_2	D_3	Supply
S_1	15	10	12	170
S_2	11	18	15	250
S_3	13	20	16	130
S_4	10	17	14	350
Demand	200	300	500	

Table 2: IBFS of the unbalanced TP according to LCM

	D_1	D_2	D_3	Supply
S_1	15 ×	10 **❸170**	12 ×	~~170~~
S_2	11 ×	18 ×	15 **❺250**	~~250~~
S_3	13 ×	20 **❻130**	16 **❼0**	~~130~~
S_4	10 **❷100**	17 ×	14 **❹250**	~~350~~, ~~250~~
Dummy (S_5)	0 **❶100**	0 ×	0 ×	100
Demand	~~200~~, ~~100~~	~~300~~, ~~130~~	~~500~~, ~~250~~	

From Table 1, it is observed that, total demand, $\sum_{j=1}^{3} b_j = 1000$ which is exceeded from total supply, $\sum_{i=1}^{4} a_i = 900$ with an amount of 100 units of the commodity. That is problem is treated as unbalanced TP. So, according to

LCM we introduce a dummy origin (S_5) with amount 100 commodity and set zero corresponding to each dummy. The allocation procedures of LC M approach and IBFS are displayed in Table 2, where ❶, ❷, ❸ etc. denote the steps of allocation.

It is observed in Table 2 that, the LCM algorithm allocates 100 units of commodities at first step of allocation procedures and eventually algorithm bound to allocate a large amount (130) correspond to expensive route.

Therefore, according to LCM method IBFS of the total transportation cost is:

$$Z = \sum_{i=1}^{5}\sum_{j=1}^{3} c_{ij}\, x_{ij} = 12550$$

Table 3: IBFS of the unbalanced TP according to proposed MLCM

	D_1	D_2	D_3	Supply
S_1	15 ×	10 **❷170**	12 ×	170
S_2	11 ×	18 ×	15 **❹250**	250
S_3	13 ×	20 **❻30**	16 **❺100**	130, 30
S_4	10 **❶200**	17 ×	14 **❸150**	350, 150
Dummy (S_5)	171 ×	171 **❼100**	171 ×	100
Demand	200	300, 130	500, 250	

Now we have performed experiment upon this unbalanced TP by the proposed MLCM approach. According to the first step of proposed algorithm, it finds amount 100 units of deficiency product according to total demands. So algorithm needs to introduce a dummy origin (S_5) with amount 100 dummy supplies. Now set 171 (as Eq. 5) as transportation cost to each and every dummy route (cells) (see Table 3). The flow of allocations as well as IBFS of the unbalanced TP is shown in a compact form in Table 3.

In Table 3, it is observed that, the algorithm first allocates to a non-dummy route but cheaper transportation cost namely 10. Finally the algorithm allocated the dummy amount (namely 100) to dummy route.

Therefore, the total transportation cost of the proposed MLCM method is

$$Z = \sum_{i=1}^{4}\sum_{j=1}^{3} c_{ij}\, x_{ij} = 11750$$

Now we have to check for its optimality. An optimal solution is one where there is no other set of roots that will further reduce the total cost.

Perform Optimality Test: Formulate an optimality test to find whether the obtained feasible solution is optimal or not. Here, number of allocations is equal to $(m+n-1) = 4+4-1 = 7$, hence optimality test can be performed. According to the MODI the first step of IBFS of the problem is shown in Table 4.

Table 4: First step of MODI for IBFS of MLCM approach

Iteration 1(T. Cost including dummy route =28850)			Objective Value = 11750		
Name		*D_1*	*D_2*	*D_3*	*Supply*
		v_1= 2	v_2= 10	v_3= 6	
S_1	u_1= 0	15 -13	10 0 **170**	12 -6	170
S_2	u_2= 9	11 0	18 1	15 0 **250**	250
S_3	u_3= 10	13 -1	20 0 **30**	16 0 **100**	130
S_4	u_4= 8	10 0 **200**	17 1	14 0 **150**	350
Dummy	u_5= 161	171 -8	171 0 **100**	171 -4	100
Demand		200	300	500	

From the Table 4, for the empty cell (2, 2), (4, 2) the value of $(u_i + v_j) - c_{ij} > 0$, the solution is not optimal. Thus we have the second step which is shown in Table 5. From the Table 5, for each empty cell difference of implicit cost $(u_i + v_j)$ and actual cost c_{ij} all are less than equal to zero i.e., $(u_i + v_j) - c_{ij} \le 0$. Then by the application of complementary slackness theorem it can be shown that the corresponding solution is optimal.

Table 5: Second MODI for IBFS of MLCM approach

Iteration 2: (T. Cost including dummy route =28820)					Objective Value = 11720			
Name			*D1*		*D2*		*D3*	*Supply*
			v_1= 3		v_2= 10		v_3= 7	
S1	u_1= 0		15		10		12	170
		-12		0	**170**	-5		
S2	u_2= 8		11		18		15	250
		0		0	**30**	0	**220**	
S3	u_3= 9		13		20		16	130
		-1		-1		0	**130**	
S4	u_4 = 7		10		17		14	350
		0	**200**	0		0	**150**	
Dummy	u_5= 161		171		171		171	100
		-7		0	**100**	-3		
Demand			200		300		500	

Table 6: The comparison between LCM, VAM and MLCM approach in unbalanced TP

Method	Total cost for IBFS	Number of iteration to obtain optimal solution
LCM	12550	3
VAM	12020	2
MLCM	11750	2

Now we have compared this experimental result of MLCM with both the existing approaches namely LCM and VAM. The comparison is shown in Table 6. From Table 6, it is observed that, the proposed MLCM approach gives better result in total transportation cost compared to the LCM and VAM approach. Moreover, number of iterations needed to obtain the optimal solution in MLCM is less than LCM approach.

6. FURTHER NUMERICAL ILLUSTRATIONS

Now we have carried out further illustrations to justify and effectiveness of the proposed algorithm. The illustrations results are displayed in Table 7. In Table 7, 20 instances are considered which are randomly generated. In the table (Iter) denotes the number of iteration required to obtain optimal solution which shown in the last column of the table. It is observed that each and every instance the proposed modified algorithm outperforms compared to LCM approach both quality of IBFSs as well as number of iterations required to obtain optimal solution. Moreover, in six instances namely Ex. 01, 02, 03, 05, 06 and 08, the IBFSs obtained by MLCM are optimal. It is also observed that in nine instances namely Ex. 01, 02, 03, 05, 06, 08 and 13 the IBFS obtained by proposed MLCM are either better or at least equal to equal that of VAM method. It also notice on the table that in some instances though the IBFS obtained by MLCM approach are worse but it required less number of iterations compared to that of VAM approach. In few instances the proposed approach performs worse compare to VAM approach. But it is known that VAM approach needs much more computation time for obtained IBFS. From these numerical illustrations it may conclude that proposed modified LCM approach is much better compared to LCM approach in the sense of finding IBFS as well as number of iterations required to obtained optimal solution. Moreover the proposed approach is comparable with VAM in the sense of finding IBFS as well as number of iteration required to obtained optimal solution. But in the sense of computational time the proposed algorithm outperforms compared to both LCM as well as VAM methods.

It is worthwhile to mention here that the existing as well as modified LCM approach is very simple. Moreover the computational cost of LCM as well as proposed MLCM is significantly cheaper than VAM and other approaches mentioned here. Though our intension of this research work is to improve LCM approach for unbalanced TP, we have considered some instances for unbalanced TP collected from different papers. The comparison results are shown in Table 8. It is noted that in the Table 8, Ref. Method indicates the method used in Reference paper pointed out in the first column of the table.

It is observed in Table 8 that, our proposed method is always significantly better than the existing LCM method. Moreover, the proposed MLCM approach is better than that of Girmay and Sharma (2013), Kulkarni & Datar (2010) and identical with other approaches namely Geetha and Anandhi (2018), Juman and Nawarathne (2019). From the table it is also clear that the proposed method shows better or at least identical solution compared to VAM approach. It is also observed in the table that the solution obtained by the proposed method is either optimal or closer to optimal solution.

Table 7: Comparison among LCM, VAM and Proposed MLCM approaches for some randomly generated unbalanced TP

Ex	Problems	LCM (Iter)	VAM (Iter)	MLCM (Iter)	Optimal Solution
01.	C_{ij} = {(15,25,35), (25,35,45), (45,55,65),(65,75,85)} S = (200,250,300,350);D = (300,400,500)	59500 (2)	57500 (1)	**57500 (1)**	57500
02.	C_{ij} = {(4,10,14), (12,19,21), (15,14,17)} S = (50,50,50); D = (30,40,55)	1835 (3)	1735 (2)	**1635 (1)**	1635
03.	C_{ij} = {(3 ,1 ,7 ,4),(2,6,5,9),(8,3,3,2)} S=(300,400,270); D=(250,350,290,450)	3490 (4)	2090 (1)	**2090 (1)**	2090
04.	C_{ij} = {(5 ,7 ,9 ,6),(6,7,10,5),(7,6,8,1)} S=(12,14,20); D=(10,6,8,12)	182 (4)	176 (3)	**172 (2)**	168
05.	C_{ij} = {(4,3,4),(10,7,5),(8,8,3),(5,6,6)} S=(11,12,10,22); D=(16,10,14)	194 (4)	162 (2)	**159 (1)**	159
06.	C_{ij} = {(45,52,63,57),(58,48,56,54),(52,55,62,58),(65,48,44,54)} S=(1350,1120,1280,1080); D=(1160,1250,1300,1565)	244120 (3)	238780 (2)	**237900 (1)**	237900
07.	C_{ij} = {(7,5,9,11),(4,3,8,6),(3,8,10,5),(2,6,7,3)} S=(35,25,20,45) ; D=(25,30,20,15)	400 (5)	360 (3)	365 (**2**)	350
08.	C_{ij} = {(4,3,5),(6,5,4),(8,10,7)} S=(90,80,70) ; D=(110,120,130)	1140 (2)	1110 (2)	**1100 (1)**	1100
09.	C_{ij} = (12,10,15,11,20),(21,17,30,14,25),(13,27,11,26,28)} S=(600,400,500) ; D=(150,280,320,200,300)	18450 (5)	**17160 (2)**	17850 (3)	17010
10.	C_{ij} = {(2,8,7,3,12),(9,5,6,10,3),(11,4,15,8,8),(10,12,6,20,7)} S=(250,450,470,330) ; D=(200,280,320,240,250)	7990 (5)	5860 (2)	**5860 (2)**	5860
11.	C_{ij} = {(8,6,9,10,12,3),(11,9,12,20,7,15),(10,18,4,8,14,2), (20,3,9,4,7,13),(9,6,16,18,12,15)} S=(550,600,300,400,650) ;D=(250,350,450,550,650,750)	19250 (6)	**16450** (5)	17350 **(3)**	13200
12.	C_{ij} = {(10,15,20,25,30,35,40),(29,45,25,30,45,55,35), (15,20,25,35,40,45,50), (18,55,43,53,34,26,33), (22,33,40,60, 65,50,39), (33,27,20,30,44,29,21), (28,31,26,35,43,38,34)} S=(1200,1350,1400,1700,1650,1250,1450) D=(1150,1500,1450,1050,1320,1180,1350)	282910 (10)	**236910** (7)	250620 **(7)**	222110
13.	C_{ij} = {(8,8,2,10,2),(11,4,10,9,4), (5,2,2,11,10), (10,6,6,5,2), (8,11,8,6,4)} S=(40,70,35,90,85) ; D=(50,55,60,70,45)	1595 (5)	1310 (4)	**1275 (4)**	1160
14.	C_{ij} = {(73,40,9,79,20),(62,93,96,8,13), (96,65,80,50,65), (57,58,29,12,87),(56,23,87,18,12)} S=(80,70,90,30,40) ;D=(60,85,65,55,95)	9580 (3)	**8860** (5)	9560 **(3)**	8710
15.	C_{ij} = {(10,15,18,37,35,13,15,17),(14,42,36,64,56,65,25,52), (76,67,54,45,53,35,32,23), (11,22,33,77,44,55,56,65), (77,18,38,68,38,68,48,28),(10,20,50,30,70,60,90,30), (70,85,75,65,55,45,35,25),(21,31,41,51,61,71,81,91)} S=(380,480,280,520,420,220,200,480) ; D=(300,400,500,600,450,550,350,330)	97030 (11)	**85740 (7)**	91930 (8)	79380
16.	C_{ij} = {(16,10,25,24,30,22),(12,12,15,13,14,15), (15,18,16,8,22,20), (7,23,20,16,18,12), (13,19,13,27,17,30), (11,15,9,33,25,10), (8,9,11,15,10,15)} S=(25,35,45,65,55,75,60) ; D=(60,30,70,50,40,45)	3540 (6)	**2775 (3)**	2925 (5)	2740
17.	C_{ij} = {(73,40,9,79,20),(62,93,96,8,13), (96,65,80,50,65), (57,58,29,12,87),(56,23,87,18,12)} S=(80,70,90,30,40) ; D=(60,85,65,55,95)	9580 (3)	**8860** (5)	9560 **(3)**	8710
18.	C_{ij} = {(20,30,25,35,40),(25,35,40,22,30), (15,25,55,20,35)} S=(500,600,400) ; D=(220,250,280,300,350)	35190 (2)	**34450 (2)**	34690 (3)	34150
19.	C_{ij} = {(20,30,40,50),(35,45,55,65), (12,24,36,48),(15,45,60,30)} S=(1250,1300,1120,1180) ; D=(1800,1200,1500,1000)	162390 (4)	**145640 (1)**	148140 (3)	145640
20.	C_{ij} = {(2,5,6,3,4),(9,1,3,2,7),(4,10,8,12,8),(3,4,5,6,13)} S=(65,40,30,75) ; D=(35,45,30,55,70)	1155 (6)	**795 (2)**	955 (5)	760

Table 8: Comparison for some referred instances regarding unbalanced TP.

Reference Paper	Unbalanced TPs	Ref. Method	LCM	VAM	MLCM	Optimal Solution
Girmay and Sharma (2013)	C_{ij} = {(5,8,6,6,3), (4,7,7,6,5), (8,4,6,6,4)} S = (800,500,900); D = (400,400,500,400,800)	9500	9800	9200	**9200**	9200
Geetha and Anandhi (2018)	C_{ij} = {(6,1,9,3), (11,5,2,8), (10,12,4,7)} S = (70,55,70); D = (85,35,50,45)	965	965	1010	965	960
Geetha and Anandhi (2018)	C_{ij} = {(7,8,11,10),(10,12,5,4),(6,11,10,9)} S=(30,45,35); D=(20,28,19,33)	606	630	630	**606**	606
Kulkarni & Datar (2010)	C_{ij} = {(3,4,6),(7,3,8),(6,4,5),(7,5,2)} S=(100,80,90,120); D=(110,110,60)	1210	1210	880	**840**	840
Juman and Nawarathne (2019)	C_{ij} = {(3,4,6),(7,3,8),(6,4,5),(7,5,2)} S=(100,80,90,120); D=(110,110,60)	840	1210	880	**840**	840

7. CONCLUSIONS

In this article, a new algorithm is developed for unbalanced transportation problem on an initial solution that gives another pattern on solutions with a more proficient way. The proposed algorithm is able to overcome the shortcoming of traditional LCM in the case of the unbalanced transportation problem. It gives results exactly or even lesser or slightly more than that of LCM & VAM methods. Furthermore, it requires a minimum number of iterations to find the optimal solution compared to existing traditional methods. In a straightforward attempt on providing a new process for obtaining IBFS for unbalanced transportation problem, we may suggest MLCM which is more efficient and effective compared to existing methods.

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