ICMIEE-PI-140240

Finite Element Analysis of Convective Heat and Mass Transfer Flow through a Channel with Heat Sources and Chemical Reaction Effect

Salma Parvin, Rehena Nasrin and M.A. Alim*
Department of Mathematics, Bangladesh University of Engineering and Technology, Dhaka-1000, Bangladesh

ABSTRACT
Present study performs the heat and mass transfer analysis for laminar double-diffusive mixed convection flow in a parallel plate reactor with four heated cylinders. An external fluid flow enters from the left inlet and exits from the right. After entering the reactor, the fluid passes four heated cylinder. Two-dimensional continuity, momentum, energy and concentration equations govern the developed mathematical model. The governing non-dimensional equations are solved by using Galerkin finite element method with triangular grid discretization system. Numerical simulations are carried out for different combinations of the heat source parameter and results are presented in terms of streamlines, temperature and concentration distributions. The results indicate that the average Nusselt and Sherwood numbers at the heat and contaminant sources strongly depend on the heat source.

Keywords: Double diffusive mixed convection, parallel plate reactor, finite element method.

1. Introduction

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, in chemical reaction engineering heat and mass transfer occur simultaneously.

Brown and Lai [1] numerically examined combined heat and mass transfer from a horizontal channel with an open cavity heated from below. Since heat and contaminant sources usually co-exist indoors, the present work is to numerically study the double-diffusive mixed convection in a vented cavity due to the discrete heat and contaminant sources. Parvin et al. [2] analyzed numerically the effect of double-diffusive natural convection of a water–Al_2O_3 nanofluid in a partially heated enclosure with Soret and Dufour coefficients. Azad et al. [3] investigated double diffusive mixed convection in an open channel with a circular heater on the bottom wall. They found that, average Nusselt number at the heat source decreases and overall mass transfer rate in terms of average Sherwood number increases with the rising of Lewis number. Muthucumaraswamy and Ganesan [4] studied effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate. Das et at. [5] studied the effect of the first order homogeneous chemical reaction on the process of an unsteady flow past an infinite vertical plate with a constant heat and mass transfer. Chamkha [6] studied the MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. He assumed that the plate is embedded in a uniform porous medium and moves with a constant velocity in the flow direction in the presence of a transverse magnetic field. Ibrahim et al. [7] have studied the effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction. Kesavaiah et al [8] have studied the effect of the chemical reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. Heat and mass transport in tubular packed reactors at reacting and non-reacting conditions was analyzed by Koning [9] where the most common models of wall-cooled tubular packed bed reactors were presented. The two dimensional axial plug flow model was used for a water gas shift reactor to compare heat conduction or mass diffusion with convective effect. Kugai [10] studied Heat and Mass Transfer in Fixed-bed Tubular Reactor. The two dimensional axial plug flow model was used for a water gas shift reactor to compare heat conduction or mass diffusion with convective effect in his study.

The objective of the present work is to investigate the effect of heat generation on the characteristics of the flow and heat/contaminant transport mechanism inside a chemical reactor channel in terms of streamlines, isotherms and iso-concentration lines.

2. Analysis

2.1. Physical Model

The domain under analysis is, as sketched in Figure 1(a)-(b), a two-dimensional cross section of a reactor channel of length L and height H with four heated tubes each of radius r, suffering the influence of a gravitational field. The centers of the heaters are located at (L/5, H/2), (2L/5, H/2), (3L/5, H/2) and (4L/5, H/2). The heaters are maintained at constant and uniform temperature T_h and concentration c_h. The air flow is entering from the left with velocity U_i, temperature T_i

* Corresponding author. Tel.: +88-01712085970
E-mail address: E-mail: salpar@math.buet.ac.bd

and concentration c_i, then passes the tubes and then the polluted hot air exhausted from the outlet opening at the right.

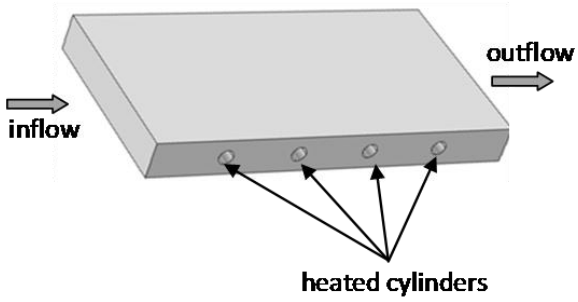

Fig-1(a). 3-D geometry of the considered reactor

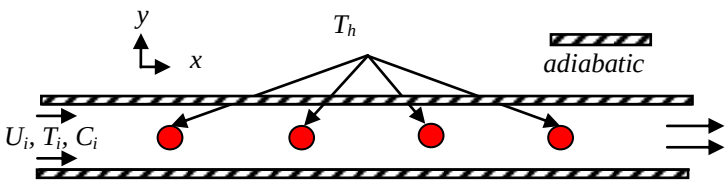

Fig-1(b). Schematic diagram of the problem

2.2. Mathematical Model

The governing mass, momentum, energy and species conservation equations have been presented by Deng et al. [11] for double-diffusive mixed convective flows driven by the combined effect of the internal buoyancy induced from temperature and concentration differences and the external mechanical driven forced flow from the inlet port. With use of the Boussinesq approximation, the dimensionless governing equations under steady-state condition are given by:

$$\frac{\partial U}{\partial X}+\frac{\partial V}{\partial Y}=0$$

$$U\frac{\partial U}{\partial X}+V\frac{\partial U}{\partial Y}=-\frac{\partial P}{\partial X}+\frac{1}{Re}\left(\frac{\partial^2 U}{\partial X^2}+\frac{\partial^2 U}{\partial Y^2}\right)$$

$$U\frac{\partial V}{\partial X}+V\frac{\partial V}{\partial Y}=-\frac{\partial P}{\partial Y}+\frac{1}{Re}\left(\frac{\partial^2 V}{\partial X^2}+\frac{\partial^2 V}{\partial Y^2}\right)+Ri\left(\theta+NC\right)$$

$$U\frac{\partial \theta}{\partial X}+V\frac{\partial \theta}{\partial Y}=\frac{1}{RePr}\left(\frac{\partial^2 \theta}{\partial X^2}+\frac{\partial^2 \theta}{\partial Y^2}\right)-H\theta$$

$$U\frac{\partial C}{\partial X}+V\frac{\partial C}{\partial Y}=\frac{1}{RePrLe}\left(\frac{\partial^2 C}{\partial X^2}+\frac{\partial^2 C}{\partial Y^2}\right)-KC$$

The above equations are non-dimensionalized by using the following dimensionless variables

$$X=\frac{x}{L},\ Y=\frac{y}{L},\ U=\frac{u}{Ui},\ V=\frac{v}{Ui},\ P=\frac{p}{\rho U_i^{\,2}},$$

$$\theta=\frac{T-T_i}{T_h-T_i},\ C=\frac{c-C_i}{\Delta c}$$

and the dimensionless parameters are Reynolds number (*Re*), Grashof number (*Gr*), Richardson number (*Ri*), Prandtl number (*Pr*), Lewis number (*Le*), the buoyancy ratio (*N*) and Chemical reaction parameter (*K*) and they are defined as follows:

$$Re=\frac{U_i L}{\nu}\ ,\ Gr=\frac{g\beta_T\left(T_h-T_i\right)L^3}{\nu^2}\ ,\ Ri=\frac{Gr}{Re^2}\ ,$$

$$Pr=\frac{\nu}{\alpha}\ ,\ Le=\frac{\alpha}{D}\ ,\ N=\frac{\beta_c\Delta c}{\beta_T\left(T_h-T_i\right)}\ ,\ K=\frac{kL^2}{D}\ ,$$

$$H=\frac{QL^2}{\alpha}$$

where ν , α , *D*, *k* and *Q* are kinematic viscosity, thermal diffusivity, solutal diffusivity, reaction coefficient and strength of heat generating source respectively. The buoyancy ratio measures the relative importance of solute and thermal diffusion in creating the density difference to drive the flow. It is clear that *N* is zero for pure thermally driven flows and infinity for pure solute driven flows.

The boundary conditions are

at the inlet: $U=1,\ V=\theta=C=0$

at the circular tube walls: $\theta=1,\ \frac{\partial C}{\partial n}=0$

at other surfaces: $\frac{\partial \theta}{\partial n}=0,\ \frac{\partial C}{\partial n}=0$

at all solid boundaries: $U=V=0$

The average Nusselt and Sherwood number may be expressed as

$$Nu=-\frac{1}{S}\int_0^S\sqrt{\left(\frac{\partial\theta}{\partial X}\right)^2+\left(\frac{\partial\theta}{\partial Y}\right)^2}\,dS \quad \text{and}$$

$$Sh=-\frac{1}{S}\int_0^S\sqrt{\left(\frac{\partial C}{\partial X}\right)^2+\left(\frac{\partial C}{\partial Y}\right)^2}\,dS$$

where *S* is the non-dimensional length of the heated/contaminant surface.

3. Computational methodology

Galerkin weighted residual method of finite element formulation is used to solve the governing equations for the present work. The application of this technique is well documented by Zienkiewicz and Taylor [12]. The nonlinear parametric solution technique is chosen to solve the governing equations. This approach will result in substantially fast convergence assurance. In addition,

the absolute convergence criteria are set to be 10^{-4} for velocities, energy and concentration.

4. Results and Discussion

The present investigation was carried out for heat generation parameter H (= 0, 1, 5 and 10) with Ri = 1, Re = 100, Pr = 0.7, Le = 1, N = 1, K = 1. Now in the following section, a detailed description of mixed convection with heat and mass transfer in a parallel plate reactor is given in terms of streamline, thermal and concentration contours for different H. In addition, the results for both average Nusselt and average Sherwood numbers at various H will be presented.

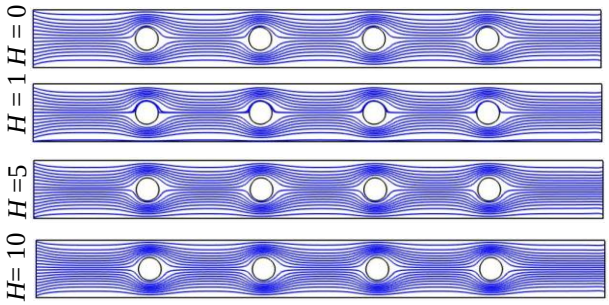

Fig-2. Effect of H on streamlines

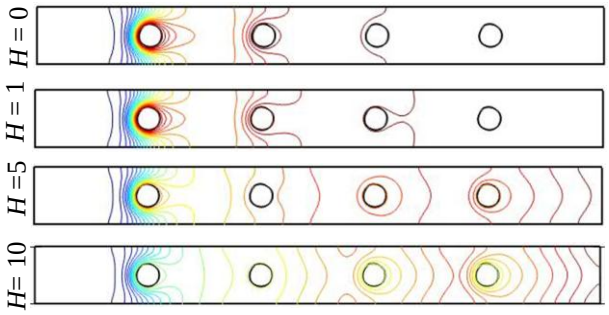

Fig-3. Effect of H on isotherms

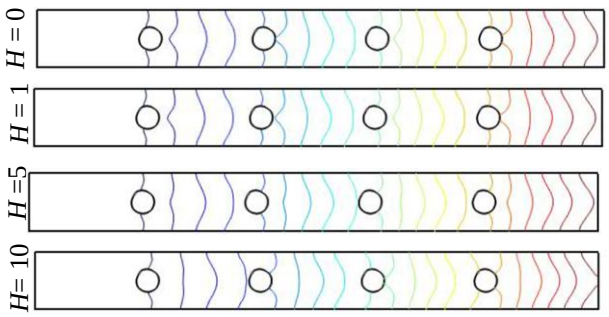

Fig-4. Effect of H on iso-concentrations

Figure 2-4 exhibits the effect of H on the streamlines, isotherms and iso-concentrations. In fact, the analysis is performed at pure mixed convection regime by fixing Ri = 1. The values of heat generation parameter are 0, 1, 5 and 10 are chosen to examine the evolution of streamline, isotherm and concentration patterns.

From Figures 2, it is observed that there is a common trend of the development of streamlines with increasing heat generation parameter. The streamlines are almost parallel to the channel wall and condensed in region between the circular heater and the channel wall. The streamlines become more condensed along the middle of the channel due to increasing heat generation effect. This indicates higher velocity.

As in Figure 3, isothermal lines have significant change due to the variation of H. variation. At H = 0, isothermal lines appear at the inlet portion of the channel. But for higher values of H, these lines spread all over the channel. It is seen from the figure that, at the highest value of H, the lower temperature lines remain at the left potion where as the higher temperature lines at the right exit port. Temperature gradient at the heat source becomes lower for increasing heat generation in the fluid. This happens because higher temperature of the fluid produces lower temperature difference between the heat source and the fluid. It is also clear that the higher temperature gradient exists at the first heater from the inlet and sequentially it reduces for the second, third and fourth.

Iso-concentration lines have also considerable change due to generating heat as shown in Figure 4. Iso-concentration lines spreads all over the channel. As heat generation increases these lines depart to the exit port which indicates higher mass transportation. This phenomenon is logical because heat generation causes higher velocity which leads to more concxentration transfer.

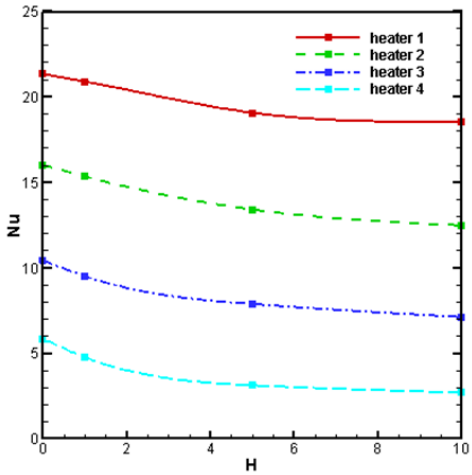

Fig-5. Effect of H on heat transfer

Figure 5 depicts the average heat transfer Nu at the four consecutive heaters for different H. Highest heat transfer rate is observed for the first heater and sequentially these values reduce for second, third and

fourth heater. This phenomenon is very logical because the flow intensity becomes lower for the last heater due to the obstacles. Increasing *H* decreases the value of *Nu* due to lowering the temperature difference

The average mass transfer *Sh* at the inlet port for different heat generation parameter is shown in Figure 6. Enhanced mass transfer rate are observed for increasing the heat generation.

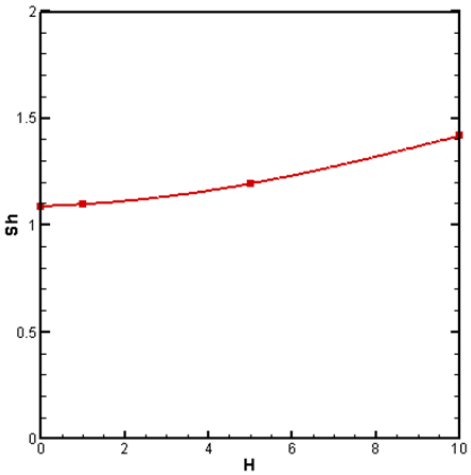

Fig-6. Effect of *H* on mass transfer

5.Conclusion

Laminar double-diffusive mixed convection flow in a parallel plate reactor with four heated cylinders for various heat generations has been studied. The following major conclusions are drawn:

- Increasing *H* has significant effects on flow, temperature and concentrations.
- Lower temperature and higher concentration gradient observe for higher *H*.
- Heat transfer reduces where as mass transfer enhance for rising values of *H*.
- The heater placed near the inlet and outlet gives respectively the highest and lowest heat transfer rate.

In general the effect of heat generation plays an important role in both heat and mass transfer for such type of reactor.

NOMENCLATURE

α : thermal diffusivity
β :expansion coefficient
ν : kinematic viscosity
ρ : density
θ : nondimensional temperature
C : nondimensional concentration
C : concentration
D : mass diffusivity
g : gravitational acceleration
Gr : Grashof number
H : height of the reactor
K : Chemical reaction parameter
L : length of the reactor
Le : Lewis number
N : buoyancy ratio
Nu : average Nusselt number
P : nondimesional pressure
Pr : Prandtl number
Re : Reynolds number
Ri :Richardson number
Sh : average Sherwood number
T : temperature
U,V : nondimensional velocity components
u,v : velocity components
X,Y : nondimensional coordinate
X,y : Cartesian coordinate

REFERENCES

[1] N. Brown and F .Lai, Correlations for combined heat and mass transfer from an open cavity in a horizontal channel, *International Communications in Heat and Mass Transfer*, Vol. 32(8), pp. 1000–1008, (2005).

[2] Salma Parvin, Rehena Nasrin, M.A. Alim, and N.F. Hossain, Double-diffusive natural convection in a partially heated enclosure using a nanofluid, *Heat Transfer—Asian Research*, 41 (6), pp 484-497, (2012).

[3] A.K. Azad, M.J.H. Munshi, M.M. Rahman, Double Diffusive Mixed Convection in a Channel with a Circular Heater , *Procedia Engineering* ,Vol. 56, pp. 157–162, (2013).

[4] R. Muthucumaraswamy, P. Ganesan, Effect of the chemical reaction and injection on flow characteristics in an unsteady upward motion of an isothermal plate, *J. Appl. Mech. Tech. Phys.*, **Vol. 42**, pp. 665-671, (2001).

[5] U.N. Das, R. Deka, V.M. Soundalgekar, Effects of mass transfer on flow past an impulsively started infinite vertical plate with constant heat flux and chemical reaction, *Forschung in Ingenieurwesen*, Vol. **60**, pp. 284-287, (1994).

[6] A.J Chamka, MHD flow of a numerical of uniformly stretched vertical permeable surface in the presence of heat generation/absorption and a chemical reaction. *Int. Comm. Heat and Mass Transf*., Vol. **30**, pp. 413-22, (2003).

[7] F.S. Ibrahim, A.M. Elaiw, A.A. Bakr, Effect of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi infinite vertical permeable moving plate with heat source and suction, *Comm. in Nonlinear Sci. and Num. Simu*l., Vol.**13**, pp. 1056-1066,(2008).

[8] DC.H. Kesavaiah, P.V. Satyanarayana and S. Venkataramana, Effects of the chemical

reaction and radiation absorption on an unsteady MHD convective heat and mass transfer flow past a semi-infinite vertical permeable moving plate embedded in a porous medium with heat source and suction. *Int.J. of Appl. Math. and Mech.*, Vol. **7**(1), pp. 52-69 (2011).

[9] B. Koning, Heat and mass transport in tubular packed bed reactors at reacting and non-reacting conditions, *Ph.D. Thesis*, University of Twente, Netherland, (2002).

[10] J. Kugai, Heat and mass transfer in fixed-bed tubular reactor, May 1st , (2008).

[11] Q. Deng, J. Zhou, C. Mei and Y. Shen, Fluid, heat and contaminant transport structures of laminar double diffusive mixed convection in a two-dimensional ventilated enclosure, *International Journal of Heat and Mass Transfer*, Vol. 47(24), pp. 5257–5269, (2004).

[12] O. Zienkiewicz and R. Taylor, *The finite element method*, Butterworth-Heinemann, 5th edition, 2000.